Chapter 6
Turbine-Based Engines: Turbojet, Turbofan, and Turboramjet Engines

6.1 Introduction

Turbine-based cycle engines (or gas turbine) together with athodyd engines represent the two main categories of reaction engines; refer to Fig. 4.1. Athodyd engines are analyzed in Chap. 5, while gas turbine engines will be analyzed in this chapter. The main difference between athodyd and turbine engines is the presence of turbomachines or rotating modules in turbine engines. There are five main types of gas turbine engines; namely: turbojet, turbofan, advanced ducted fan, turbo-ram, and turbo-rocket. In this chapter only three engines will be treated, namely, turbojet, turbofan, and turboramjet engines. Turbo-rocket engines will be analyzed in late chapters handling rockets.

Aircraft designers have always been limited by the efficiency of the available power plants. Their constant endeavor is to achieve higher thrust, lower fuel consumption, less weight, smaller frontal area, and better cooling characteristics [1].

Both piston engine and gas turbine engine are internal combustion engines. In both, air is compressed, fuel added, and the mixture ignited, and the rapid expansion of the resultant hot gas produces the power or thrust. However, combustion in a piston engine is intermittent, and the expanding gas produces shaft power through a piston and crank shaft arrangements, Chap. 4, whereas in a jet engine combustion is continuous and its power results from expanding gas being forced out of the rear of the engine as described in Chap. 5 and the present chapter.

Jet engines overcome the many drawbacks of piston engines particularly for higher-power engines. Comparison between turbine and reciprocating (piston) engines having the same power will be given hereafter [2].

Advantages of *gas turbines* over piston engines are:

Turbine engines have higher-power-to-weight ratio.
Turbine engines need less maintenance per flying hour.
Turbine engines have less frontal area and consequently less drag force.

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Turbine engines do not need any oil dilution or engine preheating before starting.
Turbine engines have lower rates of oil consumption.
Advantages of *piston engines* over gas turbines are:
Piston engines are less exposed to foreign object damage.
Piston engines have faster acceleration rates compared to turbine engines.
Piston engines have less operating temperature.
Piston engines have lower fuel consumption specially for low flight speeds.
Piston engines have much less initial costs compared to turbine engines.

6.2 Turbojet

6.2.1 Introduction

The turbojet is the basic engine of the jet age. It resembles the simplest form of gas turbine. It was separately coinvented by the two fathers of jet engines: Frank Whittle from Britain and von Ohain from Germany. The first airplane powered by a turbojet engine was the He178 German aircraft powered by the He S-3 engine on August 27, 1939. Based on von Ohain work, the German engine Junker *Jumo 004* was later developed. It was the world's first turbojet engine in production and operational use as well as the first successful axial compressor jet engine ever built. Some 8000 units were manufactured by Junkers in Germany during late World War II *which* powered several aircrafts including Messerschmitt Me 262 jet fighter, Arado Ar 234 jet reconnaissance/bomber, and prototypes of the Horten Ho 229. Frank Whittle in England having no knowledge of Ohain's engine built his W.1 turbojet engine which powered the Gloster E28/39 aircraft. Turbojets are rarely flying today due its disadvantage of high noise levels and drastic fuel consumption. Examples for the still flying turbojets powering civil transports are GE CJ610-Learjet 25 series, P&W JT12 Sabre 40, and JetStar Dash 8. Turbojet engines still power old military fighters, cruise and antiship missiles, as well as turboramjet engines. Examples for cruise missiles are AGM-109C/H/I/J/K/L and Microturbo TRI 60, while 3M-54 Klub is an example for antiship missiles and J58 turboramjet engine powering SR-71 aircraft.

As described in Chap. 5, both ramjet and scramjet engines cannot develop takeoff thrust. To overcome this disadvantage, a compressor is installed in the inlet duct so that even at zero flight speed, air could be drawn into the engine. This compressor is driven by the turbine installed downstream of the combustion chamber and connected to the compressor by a central shaft. Addition of these two rotating parts or modules (compressor and turbine) converts the ramjet into a turbojet. The three successive modules, compressor, combustion chamber, and turbine, constitute what is called *gas generator* (Fig. 6.1). The air is squeezed in the compressor to many times its normal atmospheric pressure and then forced into the combustion chamber. Fuel is sprayed into the compressed air, ignited, and burnt continuously in the combustion chamber. This raises the temperature of the

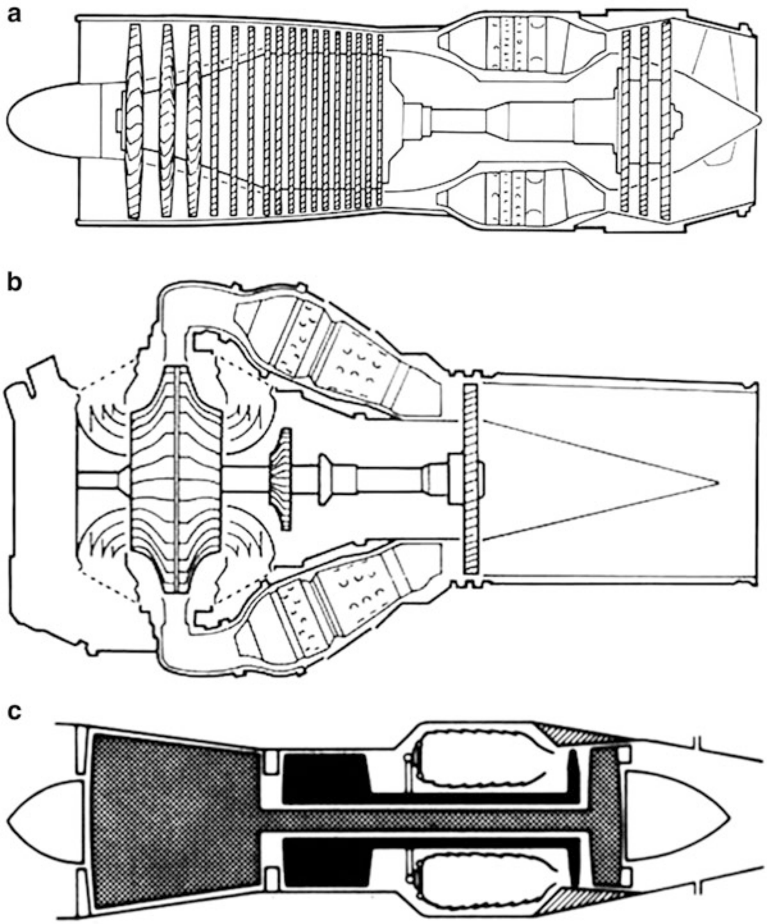


Fig. 6.1 Layout of single- and dual-spool turbojet engine. (**a**) Single-spool centrifugal flow compressor. (**b**) Single-spool axial flow compressor. (**c**) Double-spool axial flow compressor

fluid mixture to about 600°–800 ° C. The resulting burning gases expand rapidly rearward and pass through the turbine, which drives the compressor. The turbine extracts energy from the expanding gases to drive the compressor. If the turbine and compressor are efficient, the pressure at the turbine discharge will be nearly twice the atmospheric pressure, and this excess pressure is sent to the nozzle to produce a high-velocity stream of gases. These gases bounce back and shoot out of the rear of the exhaust, thus producing a thrust propelling the plane. This five-module engine (intake, compressor, combustion chamber, turbine, and nozzle) is identified as a single-spool turbojet engine.

Substantial increases in thrust can be obtained by employing an afterburner or an augmenter. It is a second combustion chamber positioned between the turbine and

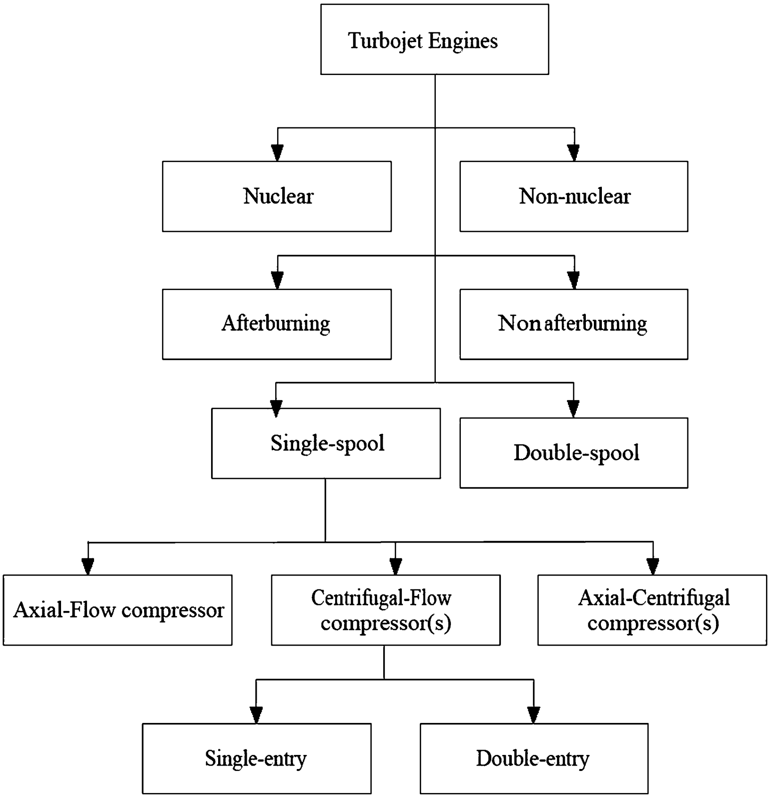


Fig. 6.2 Classification of turbojet engines

the nozzle, which is normally employed in military aircrafts. Moreover, for civil transport, the thrust force may be increased by adding a second spool (or a second set of compressor–turbine combination). Figure 6.1 illustrates two layouts for a single-spool turbojet using either an axial or centrifugal compressors while a third layout for a double-spool engine.

Classification of turbojet engines [1] is shown in Fig. 6.2. Turbojet engines may be a *nuclear* or nonnuclear engine. In the late 1940s and through the 1950s [3], preliminary work was done on developing nuclear-powered rockets and jet engines. In nuclear turbojet engine, a liquid metal is heated by the reactor which passes through a heat exchanger in the combustion chamber.

Both US and USSR nuclear programs developed the American "Crusader" NB-36, version of

B-36 [4], as well as the Russian Tupolev Tu-95 LAL [5]. Both programs were ultimately cancelled due to technical difficulties and growing safety concerns as catastrophic atomic radiation might be encountered in case of crashes/accidents of such nuclear airplanes.

Other classifications for turbojet engines may be *afterburning* or non-afterburning. Afterburning (or reheat) is a method of augmenting the basic thrust of an engine to improve the aircraft takeoff, climb, and (for military aircraft) combat performance.

Afterburner is another combustion chamber added between the turbine and the nozzle that utilizes the unburnt oxygen in the exhaust gas to support combustion. The increase in the temperature of the exhaust gas increases the velocity of the exhaust gases leaving the nozzle and therefore increases the engine thrust by 40 % at takeoff and a much larger percentage at high speeds once the plane is in the air.

A third classification of the turbojet engines is based on their *number of spools*. Turbojets may be single- or two (sometimes identified as double or dual)-spool engines. A single-spool turbojet has one shaft with a compressor and a turbine mounted at either shaft ends, while a two-spool engine has two concentric shafts as well as two compressors and turbines. The first compressor next to the intake is identified as the low-pressure compressor (LPC) which is driven by the low-pressure turbine (LPT). Both together with their shaft are denoted as the low-pressure spool. The high-pressure compressor (HPC) is situated downstream of the low-pressure compressor. Also this compressor is driven by the high-pressure turbine (HPT). Both are coupled by the high-pressure shaft (which is concentric with the low-pressure shaft) and constitute the modules of the high-pressure spool. Figure 6.1 illustrates the layout and modules of the single- and double-spool turbojet engines.

The single-spool engine may be further classified based on the *type of compressor* employed. The compression section may be composed of either a single or double compressors. Moreover, the compressor may be of the axial or centrifugal (radial) type. A single axial or centrifugal compressor is seen in some turbojets. In other cases two compressors assembled in series are found. These compressors may be either two centrifugal compressors or an axial compressor followed by a centrifugal compressor.

Lastly, this classification closes by the *type of entry* associated with the two centrifugal compressors case. Centrifugal compressors may have either a single or double entries if they were assembled in a back-to-back configuration.

6.2.2 Milestones of Turbojet Engines

Appendix B summarizes important milestones along the history of turbojet engines in both civil and military applications.

6.2.3 Thermodynamic Cycle Analysis of a Single Spool

Single-spool turbojet engine is composed of five modules, namely, intake (or inlet), compressor, combustion chamber, turbine, and nozzle. Afterburning turbojet incorporates a sixth module, namely, afterburner. Intake, combustion chamber, afterburner (if included), and nozzle are stationary elements, whereas compressor and turbine are rotating elements. Generally, in aero engines and gas turbines, two types of compressors and two types of turbines are found. These are the centrifugal and axial flow compressors (which will be analyzed in Chap. 9) and radial and axial turbines (will be analyzed in Chap. 10). Centrifugal compressor was used in both first engines of von Ohain and Whittle. The axial flow type has several stages of alternate rotating and stationary airfoil blades which can achieve compression ratios in excess of 40:1. Axial turbine consists of one or more stages of alternate stationary and rotating airfoil-section blades. The turbine extracts energy from the hot exhaust gases to drive the compressor. Radial turbines are found in small gas turbines like APU.

The characteristics of the engine that must be known in advance are flight speed (or Mach number), flight altitude, compressor pressure ratio, turbine inlet temperature (TIT) and maximum temperature of the cycle (if the afterburner is operative), type of nozzle (convergent or convergent–divergent), and fuel lowest heating value. Percentage of bled air as well as the power delivered to drive accessories and any aircraft systems must be also specified. Component efficiencies of all modules and pressure drops in stationary modules (inlet, combustor, and afterburner) must also be known. When the cycle is assumed ideal, then all efficiencies are assumed unity (or hundred percent) and all pressure drops are assumed zeros.

Figure 6.3 illustrates a layout with its designation points (Fig. 6.3a) as well as thermodynamic cycle (T-s diagram) for a single-spool turbojet with inoperative afterburner (Fig. 6.3b) and with operative afterburner (Fig. 6.3c).

The cycle different processes are described here:

–(1): air flows from far upstream (where the velocity of air relative to engine is the flight velocity) up to the intake, usually with some deceleration during cruise and acceleration during takeoff (the process is always ideal where: $h_{0a} = h_{01}$).

(1)–(2): air flows through the inlet (or intake) and ducting system up to the compressor inlet. Since this element is always considered as a diffuser, the air velocity is decreased.

(2)–(3): air is compressed in a dynamic compressor.

(3)–(4): air is heated by mixing and burring of fuel within the combustion chamber.

(4)–(5): products of combustion are expanded through a turbine to obtain power to drive the compressor.

(5)–(6): gases may or may not be further heated if the afterburner is operative or inoperative.

(6)–(7): gases are accelerated and exhausted through the exhaust nozzle.

If the engine is not fitted with afterburner, then states 5 and 6 are coincident. The amount of mass flow is usually set by flow choking in the nozzle throat.

The following remarks may be listed:

1. All components are irreversible but they are adiabatic (except burners); thus, isentropic efficiencies for the intake, compressor, turbine, and nozzle are employed.

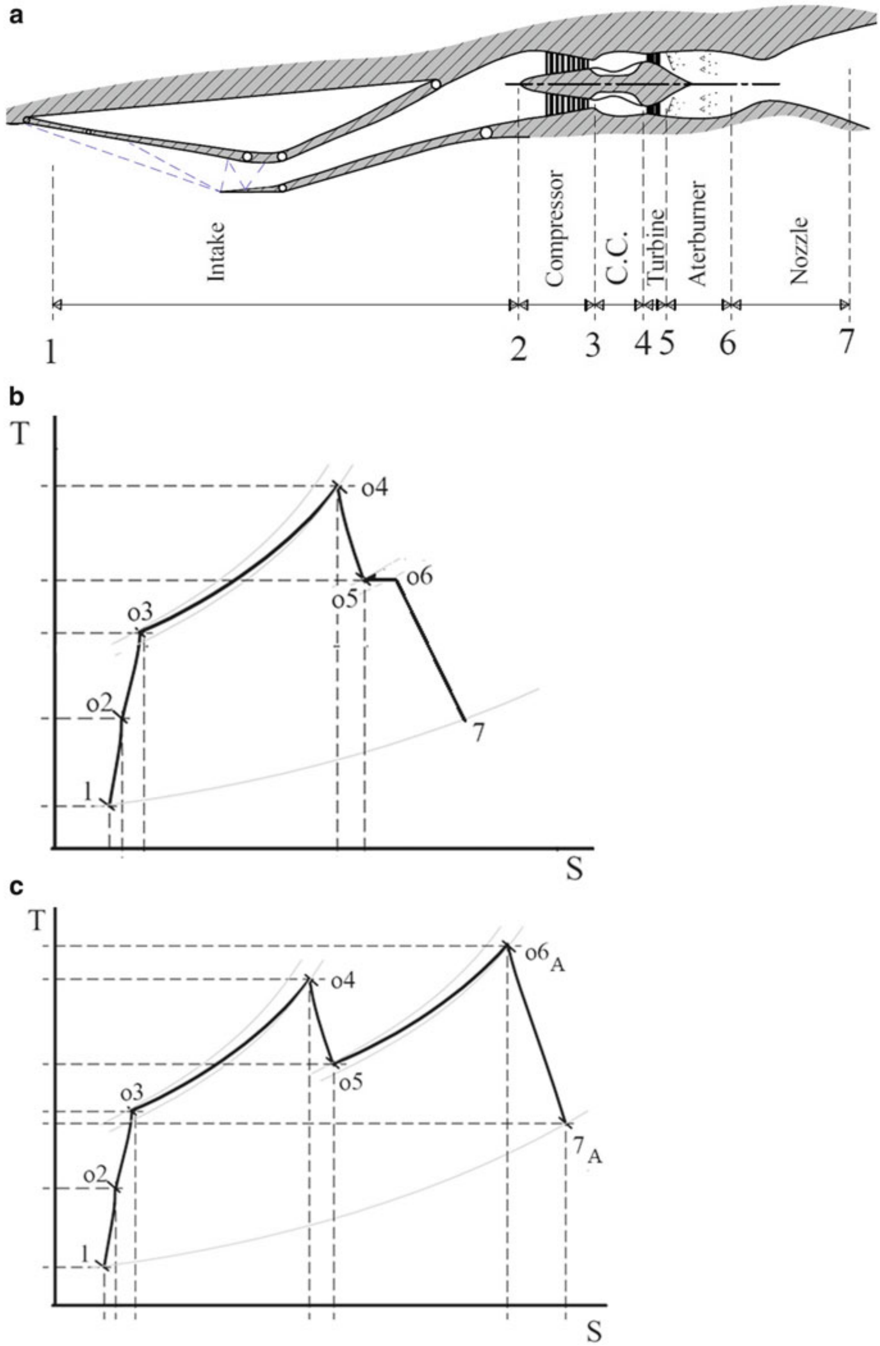


Fig. 6.3 Single-spool turbojet engine. (**a**) Schematic diagram. (**b**) T-s diagram of actual cycle with inoperative afterburner. (**c**) T-s diagram of actual cycle with operative afterburner

2. Friction in the air intake (or diffuser) reduces the total pressure from its free-stream value and increases its entropy. At the intake outlet, the total temperature is higher than the isentropic case, while the total pressure is smaller than the isentropic case which depends on the intake or diffuser efficiency (η_d). Depending on flight Mach number, the diffuser efficiency ranges as ($0.7 < \eta_d < 0.9$). The specific heat ratio ($\gamma = 1.4$). More details about intakes will be given in Chap. 8.
3. The compression of the air in the compressor increases temperature, pressure, and entropy due to irreversibilities of this process. Such increase in temperature depends on the compressor efficiency (η_c) which ranges from 0.85 to 0.90, while the specific heat ratio ($\gamma = 1.37$). A portion of the compressed air is utilized in cooling the turbine disks, blades, and the supporting bearings through an air bleed system. Thus, the air mass flow rate in the succeeding modules is somewhat smaller than that entering the compressor.
4. The burners are not simple heaters and the chemical composition of the working fluid changes during combustion process. The larger the fuel-to-air ratio (f) the greater is the deviation in the chemical composition of the products of combustion from that of the air. Losses in the combustion process are encountered due to many factors including imperfect combustion, physical characteristics of the fuel, as well as thermal losses due to conduction and radiation. Such losses are handled by introducing the burner efficiency (η_b), where: $0.97 < \eta_b \leq 0.99$. Pressure drop due to skin friction and pressure drag in the combustors (normally 3–6 % of the total pressure of the entering air) must be also taken into account.
5. The expansion process in turbine is very nearly adiabatic. However, due to friction an increase in the entropy is encountered. Moreover, the outlet temperature is higher than that of the isentropic case. Thus, the available power from the turbine is less than that in the isentropic case. The expansion process is associated with the turbine efficiency (η_t), where: $0.90 < \eta_t < 0.95$ and specific heat ratio. The specific heat ratio ($\gamma = 1.33$).
6. The afterburner is similar to the burner. This added fuel is much greater than that added in combustion chamber (burner). Losses in the afterburner are also due to imperfect combustion and thermal losses due to conduction and radiation. Such losses are handled by introducing the burner efficiency (η_{ab}), where: $0.97 < \eta_{ab} \leq 0.99$. Also a total pressure drop is encountered ranging between 3 and 6 %.
7. Finally, the expansion process in the nozzle is similar to that in the turbine and influenced by skin friction. It is also governed by the adiabatic efficiency (η_n), where: $0.95 < \eta_n < 0.98$. The specific heat ratio $\gamma = 1.36$

It is here worth mentioning that air/gas velocities within the gas generator is ignored. The velocities at the inlet to intake and outlet of nozzle are only calculated.

The different processes through the engine modules are described again here.

1. Intake

During cruise, the static pressure rises from (a) to (1) outside the intake and from (1) to (2) inside the intake. The air is decelerated relative to the engine. Since the velocity at (2) is assumed to be zero and the deceleration is adiabatic, then the total or stagnation pressure at states (0) and (1) are equal and greater than its value at state (2).

The stagnation temperatures for states (a), (1), and (2) are equal and independent from any losses:

$$T_{02} = T_{01} = T_{0a} = T_a\left(1 + \frac{\gamma - 1}{2}M^2\right) \tag{6.1}$$

Outside the engine, the total pressure remains constant; thus:

$$P_{01} = P_{0a} = P_a\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{6.2}$$

The pressure recovery within the inlet may be given; thus, the outlet pressure is obtained from the relation:

$$r_d = \frac{P_{02}}{P_{0a}} \tag{6.3a}$$

Alternatively, the efficiency (η_d) of the inlet (also denoted as intake or diffuser) is given. The outlet pressure will be given by:

$$P_{02} = P_a\left(1 + \eta_d\frac{\gamma_c - 1}{2}M_a^2\right)^{\frac{\gamma_c}{\gamma_c-1}} \tag{6.3b}$$

2. Compressor

From state (2) to (3), an irreversible adiabatic compression process takes place, which is associated with the isentropic efficiency of the compressor (η_c). Thus, with known pressure ratio (π_C), the pressure and temperature at the outlet of the compressor are evaluated from the relations:

$$P_{03} = P_{02} \times \pi_c \tag{6.4}$$

$$T_{03} = T_{02}\left(1 + \frac{\pi_c^{\frac{\gamma_c-1}{\gamma_c}} - 1}{\eta_c}\right) \tag{6.5}$$

3. Combustion chamber

In combustion process, fuel is injected in an atomized form, evaporated, and mixed with air. Spark plugs initiate the combustion process. The mass flow rate of the burnt fuel is calculated from the energy balance of the combustion chamber:

$$(\dot{m}_a + \dot{m}_f)Cp_h T_{04} = \dot{m}_a Cp_c T_{03} + \eta_b \dot{m}_f Q_R$$

The temperature at the outlet of the combustors (inlet of turbine) is determined from metallurgical limits set by the turbine blade material and is known as the turbine inlet temperature ($TIT \equiv T_{04}$) or turbine entry temperature (TET).

With $f = \dot{m}_f / \dot{m}_a$

then the fuel-to-air ratio is determined from the relation:

$$f = \frac{Cp_h T_{04} - Cp_c T_{03}}{\eta_b Q_R - Cp_h T_{04}} \tag{6.6}$$

The stagnation pressure at the outlet of combustion chamber, state (4), is less than its value at the inlet, state (3), because of fluid friction. The pressure drop is either given as a definite value or as a percentage. Thus, the outlet pressure from the combustion chamber is expressed either as:

$$P_{04} = P_{03} - \Delta P_{cc} \tag{6.7a}$$

or

$$P_{04} = P_{03}(1 - \Delta P_{\mathrm{cc}}\%) \tag{6.7b}$$

4. Turbine

The power consumed in the compressor is supplied by the turbine. If the ratio of the power needed to drive the compressor to the power available in the turbine is (λ), then the energy balance for the compressor–turbine shaft is:

$$W_c = \eta_m \ \lambda \ W_t$$

Here (λ) is of the range 75–85 %. Thus, 15–25 % of the turbine power is used in driving accessories, different actuators in the aircraft systems/cycles. The mechanical efficiency (η_m) is equal or greater than 98 %. Thus, in terms of the temperature differences:

$$\begin{gathered} Cp_c(T_{03} - T_{02}) = \lambda \eta_m (1 + f) Cp_h (T_{04} - T_{05}) \\ \left(\frac{T_{05}}{T_{04}}\right) = 1 - \frac{(Cp_c/Cp_h)T_{02}}{\lambda \eta_m (1 + f) T_{04}} \left[\left(\frac{T_{03}}{T_{02}}\right) - 1\right] \end{gathered} \tag{6.8}$$

The outlet pressure is calculated considering the adiabatic efficiency of the turbine (η_t); thus:

$$\frac{P_{05}}{P_{04}} = \left[1 - \frac{1}{\eta_t}\left(1 - \frac{T_{05}}{T_{04}}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}} \tag{6.9a}$$

Then the turbine and compressor pressure ratios are related by:

$$\left(\frac{P_{05}}{P_{04}}\right)=\left\{1-\frac{(Cp_c/Cp_h)T_{02}}{\lambda(1+f)\eta_m\eta_c\eta_t T_{04}}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma_c-1}{\gamma_c}}-1\right]\right\}^{\frac{\gamma_h}{\gamma_h-1}} \tag{6.9b}$$

$$\text{Or in general:} \qquad \pi_t=\left[1-A\left(\pi_c^{\frac{\gamma_c-1}{\gamma_c}}-1\right)\right]^{\frac{\gamma_h}{\gamma_h-1}} \tag{6.9c}$$

$$\text{Where:} \qquad A=\frac{(Cp_c/Cp_h)T_{02}}{\lambda(1+f)\eta_m\eta_c\eta_t T_{04}}$$

From equation (6.1)

$$\left(\frac{P_{05}}{P_{04}}\right)=\left\{1-\frac{(Cp_c/Cp_h)T_a}{\lambda(1+f)\eta_m\eta_c\eta_t T_{04}}\left(1+\frac{\gamma-1}{2}M^2\right)\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma_c-1}{\gamma_c}}-1\right]\right\}^{\frac{\gamma_h}{\gamma_h-1}} \tag{6.9d}$$

5. Afterburner

If the jet engine is without an afterburner, then no work or heat transfer occurs downstream of turbine, station (5). The stagnation enthalpy remains constant throughout the rest of engine. However, if there is an afterburner, we have two cases; either the afterburner is operative or inoperative.

(a) *Inoperative* afterburner

If the afterburner is inoperative (Fig. 6.3b), no further fuel is burnt, and the stagnation (total) temperature at states (5) and (6) is equal or:

$$T_{06}=T_{05} \tag{6.10}$$

Concerning the total pressure, a similar treatment to the combustion chamber is considered. Thus, based on the value of the pressure drop within the afterburner due to the skin friction and the drag from the flameholders:

$$P_{06}=P_{05}-\Delta P_{ab} \tag{6.11a}$$

$$\text{Or:} \qquad P_{06}=P_{05}(1-\Delta P_{ab}\%) \tag{6.11b}$$

(b) *Operative* afterburner

For operative afterburner (Fig. 6.3c), a subscript (A) is added to the symbols of the temperature and the pressure to denote operative afterburner. In this case an additional amount of fuel is burnt which raises the temperature to (T_{06A}):

$$T_{06A}=T_{\mathrm{MAX}} \tag{6.12}$$

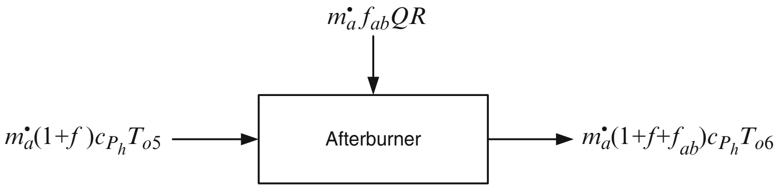


Fig. 6.4 Energy balance for afterburner

It may be higher than the turbine inlet temperature (*TIT*). The reason for such a higher temperature is that the downstream element is the nozzle, which is a nonrotating part. Thus, the walls are subjected only to thermal stresses rather than the combined thermal and mechanical stresses as in the turbine(s). The pressure at the outlet of the afterburner will be also less than its value at the inlet. It is calculated using Eq. (6.11a) and (6.11b) but P_{06A} replaces P_{06}. Conservation of mass and energy are discussed with reference to Fig. 6.4.

There is an additional fuel quantity ($\dot{m}_{\text{fab}}$) added and burnt in the afterburner. Then conservation of mass within the afterburner is expressed by:

$$\dot{m}_6 = \dot{m}_5 + \dot{m}_{\text{fab}}$$

Conservation of energy yields:

$$\dot{m}_5 h_{05} + \eta_{ab}\dot{m}_{\text{fab}} Q_R = \dot{m}_6 h_{06A}$$
$$\dot{m}_5 C_{p5} T_{05} + \eta_{ab}\dot{m}_{\text{fab}} Q_R = (\dot{m}_5 + \dot{m}_{\text{fab}}) C_{p6A} T_{06A}$$
$$(1+f) C_{p5} T_{05} + \eta_{ab} f_{ab} Q_R = (1+f+f_{ab}) C_{p6A} T_{06A}$$

The afterburner fuel-to-air ratio is calculated from the relation:

$$f_{ab} = \frac{(1+f)(Cp_{6A}T_{06A} - Cp_5 T_{05})}{\eta_{ab}Q_R - Cp_{6A}T_{06A}} \tag{6.13}$$

6. Nozzle

The two cases of inoperative and operative afterburner are again considered.

(a) Inoperative afterburner

A check for nozzle choking is performed first by calculating the critical pressure. The critical pressure is obtained from the relation:

$$\frac{P_{06}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma_n - 1}{\gamma_n + 1}\right)\right]^{\frac{\gamma_n}{\gamma_n - 1}}} \tag{6.14}$$

The critical pressure is then compared with the ambient pressure. If it is greater or equal to the ambient pressure, then the nozzle is choked ($M_7 = 1$). This means that $P_7 = P_c$ and the nozzle outlet temperature (T_7) is then calculated from the relation:

$$\left(\frac{T_{06}}{T_7}\right) = \left(\frac{\gamma_n + 1}{2}\right) \tag{6.15}$$

The exhaust velocity is then calculated as:

$$V_7 = \sqrt{\gamma_n R T_7} \tag{6.16}$$

On the other hand if the ambient pressure is greater than the critical pressure, then the nozzle is unchoked, and the exhaust pressure is equal to the ambient pressure, $P_7 = P_a$, and the exhaust gases are determined from the relation:

$$V_7 = \sqrt{2Cp_n(T_{06} - T_7)} = \sqrt{2Cp_n \eta_n T_{06}\left[1 - (P_a/P_{06})^{\frac{\gamma_n - 1}{\gamma_n}}\right]} \tag{6.17a}$$

$$\text{or} \qquad V_7 = \sqrt{\frac{2\gamma_n \eta_n R T_{06}}{(\gamma_n - 1)}\left[1 - (P_a/P_{06})^{\frac{\gamma_n - 1}{\gamma_n}}\right]} \tag{6.17b}$$

(b) Operative afterburner

The expansion process in the nozzle starts from state (06*A*) to state (7). The critical pressure is calculated from the relation:

$$\frac{P_{06A}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma_n - 1}{\gamma_n + 1}\right)\right]^{\frac{\gamma_n}{\gamma_n - 1}}} \tag{6.18}$$

If the nozzle is unchoked, the exhaust pressure will be equal to the ambient one. The jet speed will then be:

$$V_{7ab} = \sqrt{2Cp_n \eta_n T_{06A}\left[1 - (P_a/P_{06A})^{\frac{\gamma_n - 1}{\gamma_n}}\right]} \tag{6.19a}$$

If the nozzle is choked, then:

$$V_{7ab} = \sqrt{\gamma_n R T_{7A}} \tag{6.19b}$$

A general relation for the exhaust speed (whether choked or unchoked) may be obtained from Eq. (4.25) by replacing P_a by P_7; thus:

$$V_{7ab} = \sqrt{2Cp_n \eta_n T_{06A}\left[1 - (P_7/P_{06A})^{\frac{\gamma_n - 1}{\gamma_n}}\right]} \tag{6.20}$$

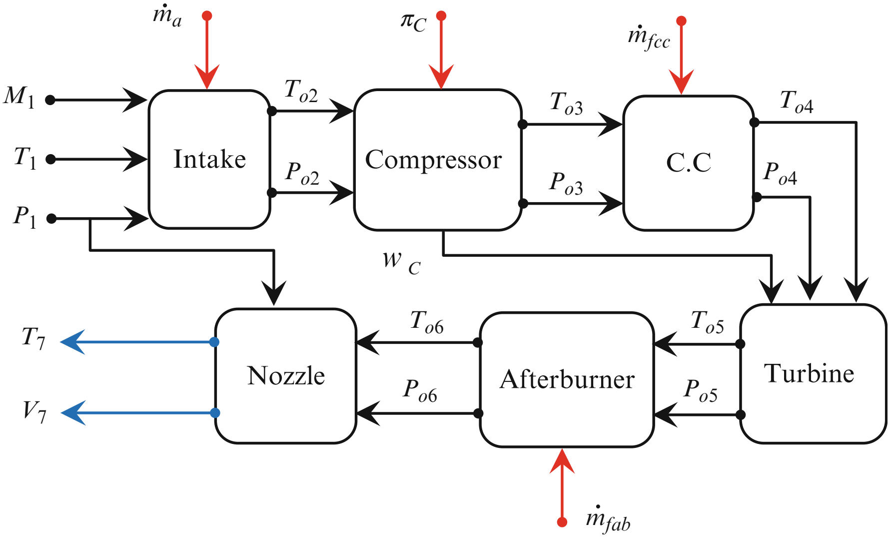


Fig. 6.5 Block diagram for the single-spool afterburning turbojet engine

The previous processes in a single-spool turbojet engines can be illustrated by the block diagram shown in Fig. 6.5 which displays flow information, input, and output data.

6.2.4 Performance Parameters of a Single Spool

The two engine parameters defining the performance of engine are the specific thrust and the specific fuel consumption. The specific thrust is expressed by the relation:

$$\frac{T}{\dot{m}_a} = [(1 + f + f_{ab})V_7 - V] + \frac{A_7}{\dot{m}_a}(P_7 - P_a) \tag{6.21}$$

The thrust specific fuel consumption (*TSFC*) is given by:

$$TSFC = \frac{\dot{m}_f + \dot{m}_{fab}}{T} \tag{6.22a}$$

Substituting from Eq. (6.21) to get:

$$TSFC = \frac{f + f_{ab}}{(1 + f + f_{ab})V_7 - V + \frac{A_7}{\dot{m}}(P_7 - P_a)} \tag{6.22b}$$

For inoperative afterburner, the same Eqs. (6.21) and (6.22) are used, but the afterburner fuel-to-air ratio f_{ab} is set equal to zero.

6.2.5 Important Definitions

Designers and operators of aero engines normally use the below definitions in their daily work.

(A) *Exhaust gas temperature gauge (EGT)*

Exhaust gas temperature is measured with a thermocouple-type pyrometer. By monitoring EGT, the pilot can get an idea of the engine's air-fuel ratio. At a stoichiometric fuel-to-air ratio, the exhaust gas temperature is different than that in a lean or rich fuel-to-air ratio. High temperatures (typically above 900 °C) can be an indicator of dangerous conditions that can lead to catastrophic engine failure.

(B) *Engine Pressure Ratio (EPR)*

Engine pressure ratio (*EPR*) is the ratio of turbine discharge to compressor inlet pressure. *EPR* is used as an indication of the amount of thrust being developed by a turbine engine. An engine pressure ratio (*EPR*) gauge is used to indicate the power output of turbojet (and turbofan) engines. Pressure measurements are recorded by probes installed in the engine inlet and at the exhaust. Once collected, the data is sent to a differential pressure transducer, which is indicated on a flight deck *EPR* gauge. *EPR* system design automatically compensates for the effects of airspeed and altitude. Changes in ambient temperature require a correction be applied to *EPR* indications to provide accurate engine power settings

(C) *Bleed*

Bleed air in aircraft engine is a compressed air that can be taken from within the engine, most often after the compressor stage(s) but before the fuel is injected in the burners. Bleed air has high temperature and high pressure (typical values are 200–250 °C and 275 kPa). This compressed air is used in aircraft in many different ways, deicing of wing leading edge, pressurizing the cabin, pneumatic actuators, starting the remaining engines, and pressurizing lavatory water storage tanks. Moreover, it is used in deicing of engine intake and cooling of turbine blades.

Example 6.1 A single-spool turbojet engine powers a military aircraft flying with a Mach number (M) at an altitude where the ambient temperature is (T_a). The compressor pressure ratio is (π) and the turbine inlet total temperature is (T_{04}). Assuming all the processes are ideal, constant air/gas properties (γ, C_p) and constant (T_{04}), and ($f \approx 0$), *prove* that the maximum thrust force is obtained when:

$$\frac{T_{04}}{T_a} = \left\{ \pi^{\frac{\gamma-1}{\gamma}} \times \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right\}^2$$

What will be the value at ground run ($M = 0$)?

Solution

Same state designation of Fig. 6.3 is used here. Air total temperature at the compressor inlet is:

$$T_{02} = T_a\left(1 + \frac{\gamma - 1}{2}M^2\right)$$

The total temperature at the compressor outlet is:

$$T_{03} = \pi^{\frac{\gamma-1}{\gamma}} T_{02}$$

Neglecting fuel-to-air ratio, assuming constant (C_p), then the energy balance between compressor and turbine yields:

$$T_{04} - T_{05} = T_{03} - T_{02}$$

$$T_{05} = T_{04} - (T_{03} - T_{02}) = T_{04} - T_{02}\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$T_{05} = T_{04} - T_a\left(1 + \frac{\gamma - 1}{2}M^2\right)\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$V_j^2 = 2Cp(T_{05} - T_7)$$

$$V_j^2 = 2Cp\left[T_{04} - T_a\left(1 + \frac{\gamma - 1}{2}M^2\right)\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) - T_7\right]$$

The ratio between turbine inlet total temperature and exhaust static temperature is:

$$\frac{T_{04}}{T_7} = \left(\frac{P_{04}}{P_7}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{03}}{P_a}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{03}P_{02}}{P_{02}\,P_a}\right)^{\frac{\gamma-1}{\gamma}} = \pi^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2}M^2\right)$$

$$V_7^2 = 2Cp\left[T_{04} - T_a\left(1 + \frac{\gamma - 1}{2}M^2\right)\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) - \frac{T_{04}}{\pi^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma-1}{2}M^2\right)}\right]$$

Denoting $\lambda = 1 + \frac{\gamma-1}{2}M^2$, then:

$$V_7^2 = 2Cp\left[T_{04} - T_a\lambda\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) - \frac{T_{04}}{\pi^{\frac{\gamma-1}{\gamma}}\lambda}\right] \qquad \text{(A)}$$

The thrust force is:

$$T = \dot{m}\,(V_7 - V)$$

The thrust is a maximum when V_7 is maximum.

Differentiate equation (A) w.r.t. compressor pressure ratio (π), then:

$$2V_7 \frac{\partial V_7}{\partial \pi} = 2Cp\left[-T_a \lambda \left(\frac{\gamma-1}{\gamma}\right)\left(\pi^{\frac{-1}{\gamma}}\right) + \left(\frac{\gamma-1}{\gamma}\right)\frac{T_{04}}{\pi^{\frac{2\gamma-1}{\gamma}}\lambda}\right]$$

For maximum thrust, then $\frac{\partial V_7}{\partial \pi} = 0$.

$$\text{Thus :} \qquad T_a\left(\frac{\gamma-1}{\gamma}\right)\left(\pi^{\frac{-1}{\gamma}}\right) = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{\lambda}\right)\left(\frac{T_{04}}{\pi^{\frac{2\gamma-1}{\gamma}}}\right)$$

$$\left(\frac{T_{04}}{T_a}\right)_{\text{Maximum thrust}} = \left(\pi^{\frac{\gamma-1}{\gamma}}\lambda\right)^2 \equiv \left[\pi^{\frac{\gamma-1}{\gamma}}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^2$$

For ground run, $M = 0$; thus:

$$\left(\frac{T_{04}}{T_a}\right)_{\text{Maximum thrust}} = (\pi)^{2\left(\frac{\gamma-1}{\gamma}\right)}$$

Example 6.2 A single-spool turbojet engine powers a military aircraft flying with a Mach number (M) at an altitude where the ambient temperature is (T_a). The compressor pressure ratio is (π) and the turbine inlet total temperature is (T_{04}). Assuming all the processes are ideal, prove that the compressor pressure ratio is expressed by the relation:

$$\pi = \left(\frac{T_{04} - \left(\frac{fQ}{Cp}\right)}{\lambda T_a}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

$$\text{Where :} \qquad \lambda = 1 + \frac{\gamma-1}{2}M^2$$

Evaluate the compressor pressure ratio for the two cases: $T_{04} = 1200$ and 1500 K:

$$\text{Assuming :} \qquad \frac{Q}{Cp} = 38,300, \; f = 0.02, \;\; M = 0.8 \; \textit{and} \; T_a = 300\,K$$

Assume ($\gamma = 1.4$) = constant.

Solution

Energy balance in combustion chamber is expressed as:

$$\dot{m}_f Q = \dot{m}_a Cp(T_{04} - T_{03})$$

Dividing by $\dot{m}_a$, then:

$$\frac{fQ}{Cp} = (T_{04} - T_{03}) \qquad \text{(A)}$$

The outlet total temperature to the compressor is:

$$T_{03} = \pi^{\left(\frac{\gamma-1}{\gamma}\right)} T_{02}$$

$$\text{Where} \qquad T_{02} = T_a\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

$$\text{Then} \qquad T_{03} = \pi^{\left(\frac{\gamma-1}{\gamma}\right)} T_a\left(1 + \frac{\gamma - 1}{2} M^2\right) \qquad \text{(B)}$$

From equations (A) and (B), then:

$$\pi = \left[\frac{T_{04} - \frac{fQ}{Cp}}{T_a\left(1 + \frac{\gamma-1}{2} M^2\right)}\right]^{\left(\frac{\gamma}{\gamma-1}\right)}$$

With $\gamma = 1.4$, $Cp = 1005$ J/kg.K:

When $T_{04} = 1200$ K, then $\pi = \left[\frac{1200 - 0.02 \times 38300}{300 \times (1 + 0.2 \times 0.64)}\right]^{3.5} = 2.389.$

When $T_{04} = 1500$ K, then $\pi = \left[\frac{1500 - 0.02 \times 38300}{300 \times (1 + 0.2 \times 0.64)}\right]^{3.5} = 15.03.$

Example 6.3 A single-spool turbojet engine has an unchoked nozzle. The fuel-to-air ratio is assumed negligible. Prove that when the thrust is a maximum, then:

$$V_e = 2V_f$$

$$\eta_p = \frac{2}{3}$$

Solution

Since the nozzle is unchoked and the fuel-to-air ratio is negligible, thrust force is expressed by the relation:

$$T = \dot{m}\left(V_e - V_f\right)$$

$$\text{Where} \qquad \dot{m} = \rho V_f A_i$$

$$\text{Then} \qquad T = \rho V_f A_i\left(V_e - V_f\right)$$

Thrust is a maximum when $\frac{\partial T}{\partial V_f} = 0$.

$$\text{Thus} \qquad \frac{\partial T}{\partial V_f} = -\rho V_f A_i + \rho A_i \left(V_e - V_f\right) = 0$$

$$-V_f + \left(V_f - V_e\right) = 0$$

$$V_e = 2V_f$$

Propulsive efficiency for unchoked and negligible fuel-to-air ratio is expressed by:

$$\eta_p = \frac{2V_f}{V_e + V_f} = \frac{2V_f}{2V_f + V_f} = \frac{2}{3}$$

Example 6.4 A single-spool turbojet engine has an unchoked nozzle.

(A) It is required to prove that:

The thrust force assuming an ideal cycle is expressed by the relation:

$$\frac{T}{P_a A_a} = \frac{2\gamma}{\gamma - 1}(\tau_r - 1)\left[(1+f)\left\{\frac{(\tau_r \tau_c \tau_t - 1)}{(\tau_r - 1)}\left(\frac{\tau_\lambda}{\tau_c \tau_r}\right)\right\}^{\frac{1}{2}} - 1\right]$$

$$\text{Where,} \qquad \tau_\lambda = \frac{T_{04}}{T_a}$$

The energy balance between turbine and compressor (neglecting the fuel-to-air ratio) yields the relation:

$$\tau_t = 1 - \frac{\tau_r(\tau_c - 1)}{\tau_\lambda}$$

The maximum thrust is attained if the temperature ratio across the compressor has the value:

$$\tau_{c_{\max}} = \frac{\sqrt{\tau_\lambda}}{\tau_r}$$

Zero thrust is generated if: $\tau_c = \frac{\tau_\lambda}{\tau_r}$

(B) Plot:

The relations $\frac{T}{P_a A_a}$ versus τ_r

The relation $\frac{T}{P_a A_a}$ versus τ_c

The temperature ratio across different modules is expressed as:

Ram: $\tau_r = \frac{T_{0a}}{T_a}$ Diffuser (intake): $\tau_d = \frac{T_{02}}{T_{0a}}$ Compressor: $\tau_c = \frac{T_{03}}{T_{02}}$

Combustion chamber: $\tau_b = \frac{T_{04}}{T_{03}}$ Turbine : $\tau_t = \frac{T_{05}}{T_{04}}$ Nozzle: $\tau_n = \frac{T_{06}}{T_{05}}$

Solution

The thrust force equation for an unchoked nozzle ($P_e = P_a$) is:

$$T = \dot{m}_a[(1+f)u_e - u_a] = \rho_a u_a^2 A_a\left[(1+f)\frac{u_e}{u_a} - 1\right]$$

$$T = \frac{P_a}{RT_a}u_a^2 A_a\left[(1+f)\frac{u_e}{u_a} - 1\right]$$

$$\frac{T}{P_a A_a} = \frac{\gamma u_a^2}{\gamma RT}\left[(1+f)\frac{u_e}{u_a} - 1\right]$$

$$\frac{T}{P_a A_a} = \gamma M_a^2\left[(1+f)\frac{u_e}{u_a} - 1\right] \qquad \text{(A)}$$

The ratio between exhaust and flight speeds may be correlated as:

$$\frac{u_e}{u_a} = \frac{M_e}{M_a}\sqrt{\frac{T_e}{T_a}} \qquad \text{(B)}$$

The pressure ratio across different modules is expressed as:

Ram: $\pi_r = \frac{P_{0a}}{P_a}$

Diffuser (intake): $\pi_d = \frac{P_{02}}{P_{0a}}$

Compressor: $\pi_c = \frac{P_{03}}{P_{02}}$

Combustion chamber: $\pi_b = \frac{P_{04}}{P_{03}}$

Turbine : $\pi_t = \frac{P_{05}}{P_{04}}$

Nozzle: $\pi_n = \frac{P_{06}}{P_{05}} = \frac{P_{0e}}{P_{05}}$

The exhaust total pressure may be expressed as:

$$P_{0e} = P_a\frac{P_{0a}P_{02}P_{03}P_{04}P_{05}P_{0e}}{P_a\, P_{0a}P_{02}P_{03}P_{04}P_{05}}$$

$$P_{0e} = P_a\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n$$

For ideal diffuser, burner, and nozzle, then:

$$\pi_d = \pi_b = \pi_n = 1$$

The exhaust total pressure can be then expressed as:

$$P_{0e} = P_a\pi_r\pi_c\pi_t = P_a\left(1 + \frac{\gamma - 1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

Since the nozzle is unchoked, then:

$$P_a = P_e$$

$$\pi_r\pi_c\pi_t = \left(1 + \tfrac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

Moreover, with:

$$\begin{aligned}
&\pi_r = \tau_r^{\frac{\gamma}{\gamma-1}}, \quad \pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}, \quad \pi_t = \tau_t^{\frac{\gamma}{\gamma-1}} \\
&P_a = P_e \\
&(\tau_r \tau_c \tau_t)^{\frac{\gamma}{\gamma-1}} = \left(1 + \tfrac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}
\end{aligned}$$

Thus:

$$\begin{aligned}
\tau_r \tau_c \tau_t &= 1 + \frac{\gamma-1}{2} M_e^2 \\
M_e^2 &= \frac{2}{\gamma-1}(\tau_r \tau_c \tau_t - 1)
\end{aligned} \qquad \text{(C)}$$

Also, ram total temperature is expressed as:

$$\begin{aligned}
\tau_r &= 1 + \frac{\gamma-1}{2} M_a^2 \\
M_a^2 &= \frac{2}{\gamma-1}(\tau_r - 1) \qquad \text{(D)}
\end{aligned}$$

Thus, the ratio between exhaust and flight Mach numbers is:

$$\frac{M_e^2}{M_a^2} = \frac{(\tau_r \tau_c \tau_t - 1)}{(\tau_r - 1)} \qquad \text{(E)}$$

The exhaust total temperature may be further expressed as:

$$\begin{aligned}
T_{0e} &= T_a \frac{T_{0a} T_{02} T_{03} T_{04} T_{05} T_{0e}}{T_a\, T_{0a} T_{02} T_{03} T_{04} T_{05}} \\
T_{0e} &= T_a \tau_r \tau_d \tau_c \tau_b \tau_t \tau_n
\end{aligned}$$

With $\tau_d = \tau_n = 1$, then from (C):

$$\begin{aligned}
T_{0e} &= T_a \tau_r \tau_c \tau_b \tau_t \\
T_{0e} &= T_e \left(1 + \frac{\gamma-1}{2} M_e^2\right) \\
T_{0e} &= T_e \tau_r \tau_c \tau_t \\
\frac{T_e}{T_a} &= \tau_b = \frac{T_{04}}{T_{03}}
\end{aligned}$$

Now, define $\tau_\lambda = \frac{T_{04}}{T_a}$.

Then:

$$\frac{T_e}{T_a} = \frac{T_{04}}{T_{03}} = \frac{T_{04}}{T_a}\frac{T_a}{T_{03}} = \frac{T_{04}}{T_a}\frac{T_{02}}{T_{03}}\frac{T_a}{T_{02}} = \frac{\tau_\lambda}{\tau_c\tau_r} \qquad \text{(F)}$$

From (A) through (F), then:

$$\frac{T}{P_aA_a} = \frac{2\gamma}{\gamma-1}(\tau_r - 1)\left[(1+f)\left\{\frac{(\tau_r\tau_c\tau_t - 1)}{(\tau_r - 1)}\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)\right\}^{\frac{1}{2}} - 1\right] \qquad \#$$

Next, assuming that the turbine provides a work just sufficient to rotate the compressor, then:

$$\dot{m}_a(T_{03} - T_{02}) = \left(\dot{m}_a + \dot{m}_f\right)(T_{04} - T_{05})$$
$$\tau_r(\tau_c - 1) = (1+f)\tau_\lambda(1 - \tau_t)$$
$$\tau_t = 1 - \frac{\tau_r(\tau_c - 1)}{(1+f)\tau_\lambda}$$

Neglecting the fuel-to-air ratio, then:

$$\tau_t = 1 - \frac{\tau_r(\tau_c - 1)}{\tau_\lambda} \qquad \text{(G)} \quad \#$$

Now, to maximize the thrust for a certain flight Mach number, from equation (A), the ratio between exhaust and flight speeds $(\frac{u_e}{u_a})$ must be maximum.

From equations (E) through (G):

$$\left(\frac{u_e}{u_a}\right)^2 = \left(\frac{M_e}{M_a}\right)^2\frac{T_e}{T_a} = \frac{(\tau_r\tau_c\tau_t - 1)}{(\tau_r - 1)}\frac{\tau_\lambda}{\tau_c\tau_r}$$
$$\left(\frac{u_e}{u_a}\right)^2 = \frac{1}{(\tau_r - 1)}\left(\tau_t\tau_\lambda - \frac{\tau_\lambda}{\tau_c\tau_r}\right) = \frac{1}{(\tau_r - 1)}\left[\left\{1 - \frac{\tau_r(\tau_c - 1)}{\tau_\lambda}\right\}\tau_\lambda - \frac{\tau_\lambda}{\tau_c\tau_r}\right]$$
$$\left(\frac{u_e}{u_a}\right)^2 = \frac{1}{(\tau_r - 1)}\left[\tau_\lambda - \tau_r(\tau_c - 1) - \frac{\tau_\lambda}{\tau_c\tau_r}\right]$$

Now, differentiate the above relation with respect to temperature ratio across the compressor:

$$\frac{\partial}{\partial\tau_c}\left(\frac{u_e}{u_a}\right)^2 = \frac{1}{(\tau_r - 1)}\left[-\tau_r + \frac{\tau_\lambda}{\tau_c^2\tau_r}\right] = 0$$
$$\tau_r = \frac{\tau_\lambda}{\tau_c^2\tau_r} \qquad \#$$
$$\tau_{c_{\max}} = \frac{\sqrt{\tau_\lambda}}{\tau_r}$$

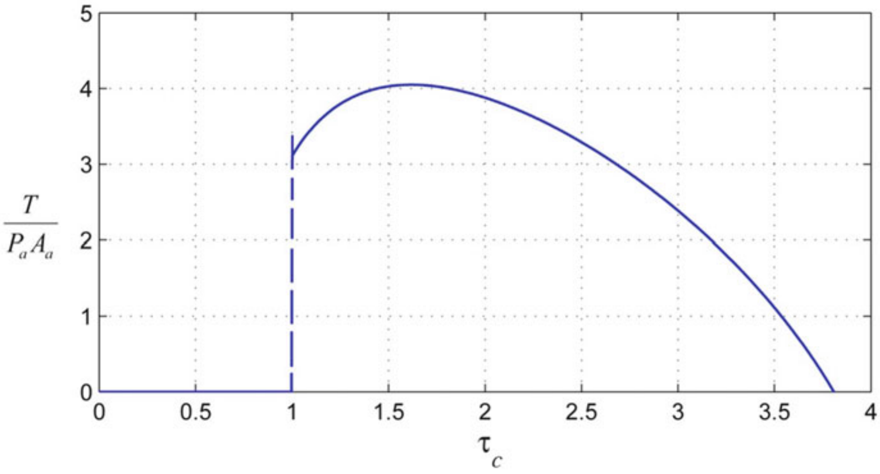


Fig. 6.6 Nondimensional thrust versus temperature ratio in compressor

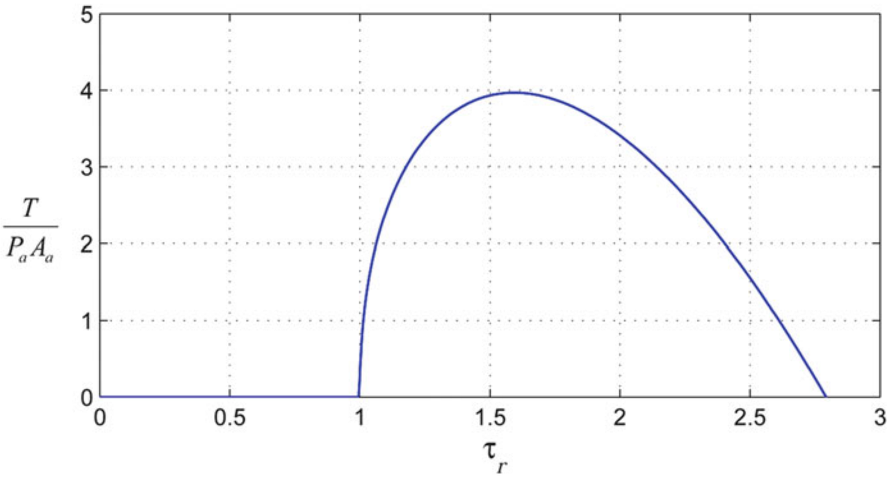


Fig. 6.6b Nondimensional thrust versus ram temperature ratio

The *thrust force attains a zero value* when $\frac{u_e}{u_a} = 1$, which is achieved when:

$$\tau_c = \frac{\tau_\lambda}{\tau_r} \qquad \#$$

Example 6.5 A civil transport aircraft powered by four turbojet engines is flying at a Mach number of 0.85 and an altitude where the ambient temperature is 230 K. The overall pressure ratio is 20 and the maximum temperature is 1500 K. Due to an *ingested bird*, one engine failed and the aircraft flew by three engines at the same altitude and Mach number. The pilot advanced the throttle lever to increase the rate of fuel flow and thus the turbine inlet temperature. This led to an increase in exhaust gas velocity to compensate for the thrust of the failed engine. The thrust force of the operative engines was thus increased. Calculate:

The original and final exhaust speeds
The new maximum total temperature
Fuel-to-air ratio in both cases (fuel heating value is 42,000 kJ/kg)

Assume that:

All processes are ideal
Constant specific heats and specific heat ratio at all engine modules, such that $Cp = 1005\ \text{J/kg/K}$ and $\gamma = 1.4$
The works of both compressor and turbine are equal
Air mass flow rate into each engine in both cases is constant
Neglect fuel mass flow rate in thrust and energy balance calculation

Solution

(A) *Four operative engines*

The flight speed is then: $u = M\sqrt{\gamma R T_a} = 258.5\ \text{m/s}$

Intake

$$T_{02} = T_a\left(1 + \frac{\gamma - 1}{2}M^2\right) = 263.24\ \text{K}$$

Compressor

The air temperature at compressor outlet is:

$$T_{03} = T_{02} \times \pi^{\frac{\gamma-1}{\gamma}} = 263.24 \times 20^{0.286} = 620\ \text{K}$$

The specific work of compressor is:

$$w = C_p(T_{03} - T_{02}) = C_p T_{02}\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) = 1005 \times 263.24\left(20^{0.286} - 1\right)$$

$w = 358.62\ \text{kN/kg}$ and $w/C_p = 256.83\ \text{kN/kg}$

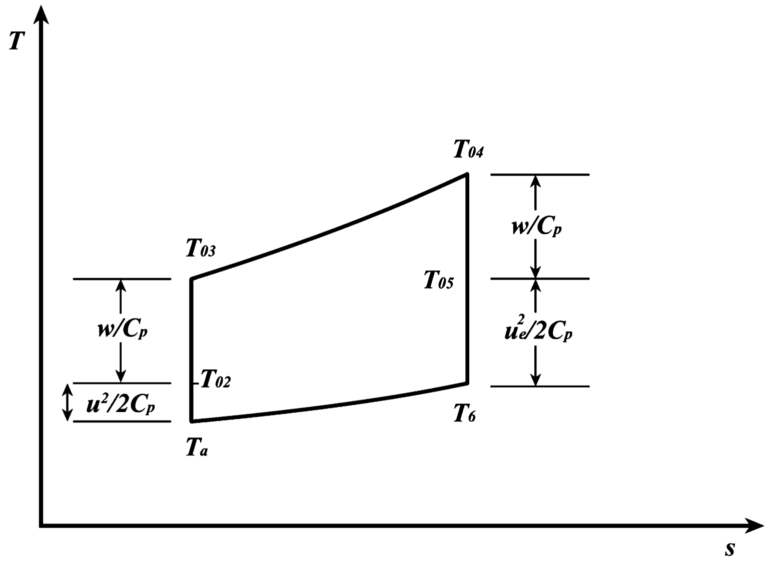


Fig. 6.7a T-s diagram for four engines operating case

Combustion chamber

The fuel-to-air ratio is:

$$f = \frac{C_p(T_{04} - T_{03})}{Q_{HV} - C_p T_{04}} = \frac{1.005(1500 - 620)}{42{,}000 - 1.005 \times 1500} = 0.02184$$

Turbine

The assumptions of constant specific heat through the engine as well as equal work for both compressor and turbine imply that both air and gas flow rates in compressor and turbine are equal (negligible fuel flow mass). Thus:

$$T_{05} = T_{0\max} - \frac{w}{C_p} = 1500 - 356.84 = 1143.17 \text{ K}$$

Nozzle

Since combustion process is ideal, then:

$$\frac{P_{04}}{P_6} = \frac{P_{03}}{P_a} = \frac{P_{03}P_{02}}{P_{02}\,P_a} = \frac{P_{03}}{P_{02}}\left(\frac{T_{02}}{T_a}\right)^{\frac{\gamma}{\gamma-1}} = \pi\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$
$$\frac{P_{04}}{P_6} = 20\left(1 + 0.2 \times 0.85^2\right)$$
$$\frac{P_{04}}{P_6} = 32.076$$

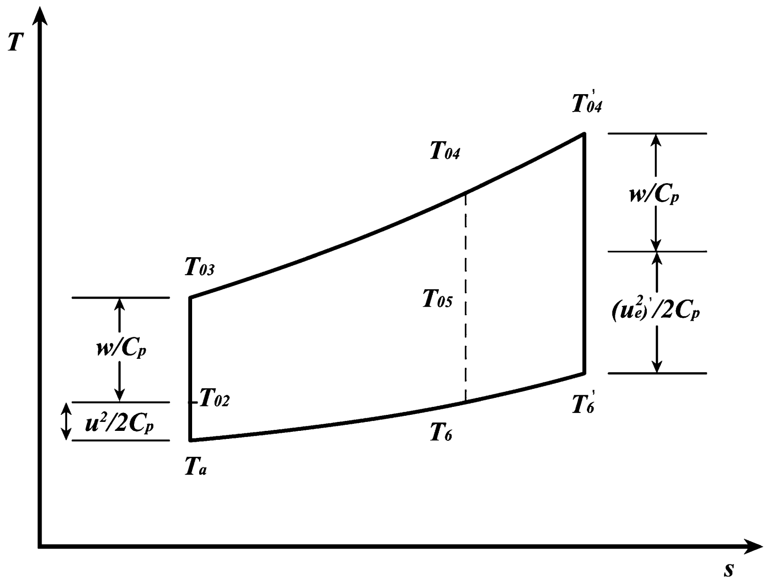


Fig. 6.7b T-s diagram for three engines operating case

$$\text{But} \qquad \frac{P_{04}}{P_{05}} = \left(\frac{T_{04}}{T_{05}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1500}{1143.17}\right)^{3.5} = 2.5878$$

$$\frac{P_{05}}{P_6} = \frac{P_{05}}{P_{04}}\frac{P_{04}}{P_6} = \frac{32.076}{2.5878} = 12.3947$$

$$\text{Also} \qquad T_6 = T_{0\max}/(32.076)^{0.286} = 1500/2.6963 = 556.32 \ \text{K}$$

The exhaust speed is then:

$$u_e = \sqrt{2C_p(T_{05} - T_6)} = \sqrt{2 \times 1005 \times (1143.17 - 556.32)}$$
$$u_e = 1086 \ \text{m/s}$$

Specific thrust is then:

$$T/\dot{m}_a = (1 + f)u_e - u \approx u_e - u = 1086 - 258.5 = 827.58 \ \text{m/s}$$

(B) *Only three engines operative*

This case represents a one engine failure; thus, aircraft is powered by three engines only.

For the same flight speed, the aircraft drag is the same which must be equal to the sum of thrust forces generated by operative engines. The pilot had to advance throttle lever to a forward setting leading to a greater fuel flow and consequently higher maximum (turbine inlet) temperature T'_{04}.

For the same compressor–turbine work as in the four-engine case, the exhaust speed is increased to u_e', such that the corresponding engine thrust, T', will be governed by the relation:

$$3T' = 4T$$

Assuming the air mass flow rate is maintained constant, thus:

$$\dot{m}'_a = \dot{m}_a$$

And:

$$\begin{aligned}
&3\dot{m}'_a(u_e' - u) = 4\dot{m}_a(u_e - u)\\
&3(u_e' - u) = 4(u_e - u)\\
&u_e' = \frac{4}{3}u_e - \frac{1}{3}u = \frac{4 \times 1086 - 258.5}{3} = 1316.9\ \text{m/s}
\end{aligned}$$

Since the compressor pressure ratio is constant and flight speed is also unchanged, then the overall pressure ratio is unchanged or:

$$\begin{aligned}
&\frac{T_{04}'}{T_6'} = \left(\frac{P_{03}}{P_a}\right)^{\frac{\gamma-1}{\gamma}} = (32.076)^{0.286} = 2.6963\\
&T_{04}' = T_6' + \frac{u_e'^2}{2C_p} + \frac{w}{C_p}
\end{aligned}$$

$$\begin{aligned}
&\frac{u_e'^2}{2C_p} + \frac{w}{C_p} = 1219.6\ \text{K}\\
&2.6963T_6' = T_6' + 1219.6\\
&T_6' = 718.92\ \text{K}\\
&T_{04}' = 1938\ \text{K}
\end{aligned}$$

The new fuel-to-air ratio is:

$$f = \frac{C_p(T_{04} - T_{03})}{Q_{HV} - C_pT_{04}} = \frac{1.005(1938 - 620)}{42{,}000 - 1.005 \times 1938} = 0.03307$$

It may be concluded that:

Fuel-to-air ratio had been drastically increased from 0.02184 to 0.03307 (52.3 %).

The maximum temperature (turbine inlet) increased from 1500 K to 1938 K (29.2 %), which most probably led to a turbine overheat.

Practically, pilots normally follow other procedures, among which the flight altitude and speed are reduced which reduce the drag force. Thus, the thrust generated by each engine in the second case is close to its value prior engine failure. In this case the maximum temperature may be kept within the range of 100–110 % of its original value.

6.2.6 Double-Spool Turbojet

The two-spool (or double-spool) engines are composed of two compressors coupled to two turbines through two shafts (or spools) (Fig. 6.8a). Double-spool engines are designed to fulfill the high-thrust requirements for both military and civil aircrafts. The first spool is composed of the low-pressure compressor (LPC) located downstream the intake. It is coupled and driven by the low-pressure turbine (LPT) located upstream of the nozzle or afterburner (if fitted). The second spool is composed of the high-pressure compressor (HPC) located downstream of the LPC and upstream of the combustion chamber. This HPC is driven by the high-pressure turbine (HPT) installed downstream of the combustion chamber and upstream of the LPT. Low-pressure spool rotates at speed (N_1) while high-pressure spool rotates at a higher speed (N_2). Also two-spool turbojet may be fitted with afterburner as shown in Fig. 6.8a. Afterburner is located downstream the LPT and upstream of the nozzle. Most afterburning turbojet engines are fitted to military airplanes. Only supersonic transports (Concorde and Tu-144) are powered afterburning turbojet engines, namely, Rolls-Royce/SNECMA Olympus 593 Mk 610 and Kolesov RD-36-51 engines. Moreover, afterburning turbojets employ variable area intake and nozzle.

It may be concluded that two-spool turbojet engines may be composed of seven or eight modules. These are intake, two compressors, combustion chamber, two turbines, and a nozzle. For afterburning engines, the eighth module is the afterburner.

6.2.7 Thermodynamic Analysis of Double-Spool Turbojet

Real (nonideal) case will be considered here, as the ideal performance can be considered as a special case where the efficiencies are set equal to unity and no pressure losses. Modules of two-spool turbojet engine were listed above.

Intake is identical to the case of single-spool turbojet. Thus, Eqs. (6.1), (6.2), and (6.3) are used and no need to be repeated here. Now the successive elements (LPC through nozzle) will be examined.

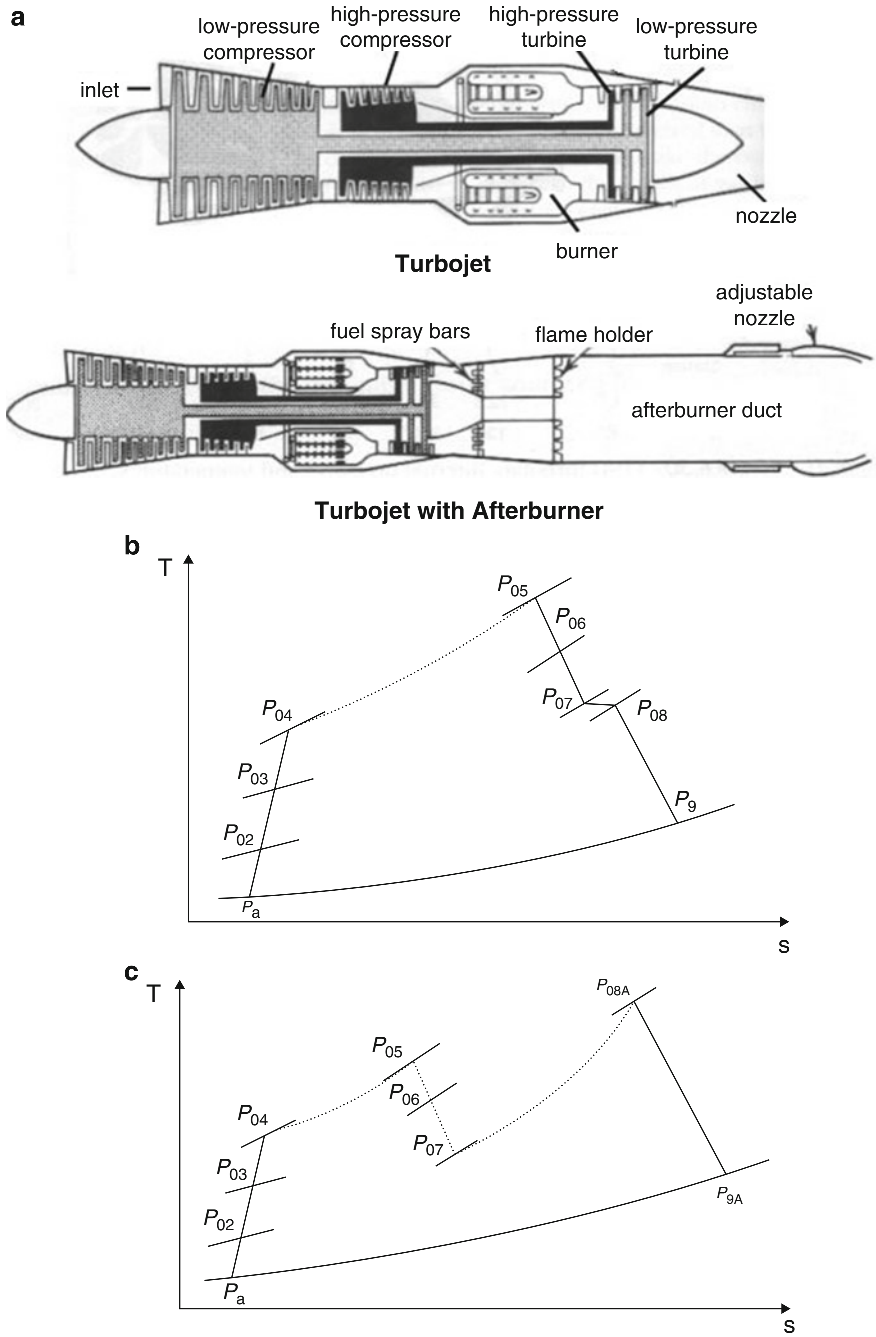


Fig. 6.8 Two (double)-spool turbojet engine. (**a**) Layout of two-spool turbojet engine (with and without afterburner). (**b**) T-s diagram for non-afterburning turbojet. (**c**) T-s diagram for afterburning turbojet

(A) Non-afterburning Engine

1. Low-pressure compressor (LPC)

For a known compressor pressure ratio (π_{c1}) and isentropic efficiency (η_{c1}), then the pressure and temperature at the outlet of the LPC are given by the following relations:

$$P_{03} = P_{02} \times \pi_{c1} \tag{6.23}$$

$$T_{03} = T_{02}\left(1 + \frac{\pi_{c1}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{c1}}\right) \tag{6.24}$$

2. High-pressure compressor (HPC)

Similarly, both of the pressure ratio (π_{c2}) and its isentropic efficiency (η_{c2}) are known. Thus, the pressure and temperature at the outlet of the HPC are given by the following relations:

$$P_{04} = (P_{03})(\pi_{c2}) \tag{6.25}$$

$$T_{04} = T_{03}\left[1 + \frac{\pi_{c2}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{c2}}\right] \tag{6.26}$$

3. Combustion chamber

The temperature at the end of combustion process T_{05} is generally known. It is the maximum temperature in the cycle if the afterburner is inoperative. A similar analysis to single-spool turbojet yields:

$$P_{05} = P_{04} - \Delta P_{cc} \tag{6.27a}$$

$$\text{Or} \qquad P_{05} = P_{04}(1 - \Delta P_{cc}\%) \tag{6.27b}$$

The energy balance for the combustion chamber yields the following relation for the fuel-to-air ratio (f):

$$\dot{m}_a(1+f)Cp_h T_{05} = \dot{m}_a Cp_c T_{04} + \eta_b \dot{m}_f Q_R$$

$$\text{With} \qquad f = \dot{m}_f/\dot{m}_a$$

$$f = \frac{(Cp_h/Cp_c)(T_{05}/T_{04}) - 1}{\eta_b(Q_R/Cp_c T_{04}) - (Cp_h/Cp_c)(T_{05}/T_{04})} \tag{6.28}$$

4. High-pressure turbine (HPT)

The power generated in the high-pressure turbine (HPT) is used in driving the high-pressure compressor (HPC) in addition to some accessories. If the ratio of the power needed to drive the HPC to the power available from the HPT is (λ_1), then the energy balance for the compressor–turbine shaft is:

$$W_{\mathrm{HPC}} = \lambda_1 \eta_{m1} W_{\mathrm{HPT}}$$

Here (λ_1) is of the range 75–85 % and (η_{m1}) is the mechanical efficiency of high-pressure spool (normally 99 + %). Thus, in terms of the temperatures differences:

$$\begin{aligned} Cp_c(T_{04} - T_{03}) &= \lambda_1(1+f)\eta_{m1}Cp_h(T_{05} - T_{06}) \\ \left(\frac{T_{06}}{T_{05}}\right) &= 1 - \frac{(Cp_c/Cp_h)T_{03}}{\lambda_1(1+f)\eta_{m1}T_{05}}\left[\left(\frac{T_{04}}{T_{03}}\right) - 1\right] \end{aligned} \tag{6.29}$$

Then the pressure ratios of the high-pressure turbine and high-pressure compressor are related by:

$$\left(\frac{P_{06}}{P_{05}}\right) = \left(1 - \frac{T_{05} - T_{06}}{\eta_{t1}T_{05}}\right)^{\frac{\gamma_t}{\gamma_t - 1}} \tag{6.30}$$

where η_{t2} is the isentropic efficiency of the high-pressure turbine.

5. Low-pressure turbine (LPT)

The power consumed in the LPC is supplied through the LPT in expansion. With (λ_2) identifying the ratio of the power needed to drive the compressor to the power available in the turbine, then the energy balance for the low-pressure spool is:

$$W_{\mathrm{LPC}} = \lambda_2 \eta_{m2} W_{\mathrm{LPT}}$$

Here (λ_2) is of the range 75–85 %. Thus, in terms of the temperatures differences:

$$\begin{aligned} Cp_c(T_{03} - T_{02}) &= \lambda_2\eta_{m2}(1+f)Cp_h(T_{06} - T_{07}) \\ \left(\frac{T_{07}}{T_{06}}\right) &= 1 - \frac{(Cp_c/Cp_h)T_{02}}{\lambda_2\eta_{m2}(1+f)T_{06}}\left[\left(\frac{T_{03}}{T_{02}}\right) - 1\right] \end{aligned} \tag{6.31}$$

Then the turbine and compressor pressure ratios are related by:

$$\left(\frac{P_{07}}{P_{06}}\right) = \left\{1 - \frac{(Cp_c/Cp_h)T_{02}}{\lambda_2\eta_{m2}(1+f)\eta_{c1}\eta_{t1}T_{06}}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right]\right\}^{\frac{\gamma_h}{\gamma_h - 1}} \tag{6.32}$$

From the diffuser part, Eq. (6.1):

$$\left(\frac{P_{07}}{P_{06}}\right) = \left\{1 - \frac{(Cp_c/Cp_h)T_a}{\lambda_2\eta_{m2}(1+f)\eta_{c1}\eta_{t1}T_{06}}\left(1 + \frac{\gamma - 1}{2}M^2\right)\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right]\right\}^{\frac{\gamma_h}{\gamma_h - 1}} \tag{6.33}$$

6. Jet pipe

The jet pipe following the low-pressure turbine and preceding the nozzle is associated with a slight pressure drop, while the total temperature remains unchanged:

$$P_{08} = P_{07} - \Delta P_{\text{jet pipe}} \tag{6.34a}$$

$$T_{08} = T_{07} \tag{6.34b}$$

7. Nozzle

First a check for nozzle choking is performed. Thus, the critical pressure is obtained from the relation:

$$\frac{P_{08}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}} \tag{6.35}$$

If the nozzle is unchoked, then the outlet pressure is equal to the ambient pressure. The jet speed is now evaluated from the relation:

$$V_9 = \sqrt{2Cp_h \eta_n T_{08}\left[1 - (P_a/P_{08})^{\frac{\gamma_h - 1}{\gamma_h}}\right]} \tag{6.36a}$$

If the nozzle is choked, then the outlet temperature (T_8) is calculated from the relation:

$$\left(\frac{T_{08}}{T_9}\right) = \left(\frac{\gamma_h + 1}{2}\right)$$

The jet speed is expressed as:

$$V_9 = \sqrt{\gamma_h R T_9} \tag{6.36b}$$

(B) Afterburning Engine

The same treatment for all the modules upstream to the afterburner is applied here. Now, for operating afterburner, the pressure at the outlet to the afterburner is determined from one of the following relations:

$$P_{08A} = P_{07} - \Delta P_{ab} \tag{6.37a}$$

$$\text{Or} \qquad P_{08A} = P_{07}(1 - \Delta P_{ab}\%) \tag{6.37b}$$

The maximum temperature in the cycle is obtained at the end of the combustion process:

$$T_{08A} = T_{\mathrm{MAX}}$$

The afterburner fuel-to-air ratio is calculated from the energy balance in the afterburner which gives:

$$f_{\mathrm{ab}} = \frac{(1+f)(Cp_{8A}T_{08A}\text{-}Cp_7T_{07})}{\eta_{ab}Q_R - Cp_{8A}T_{08A}} \tag{6.38}$$

Nozzle:

The very hot gases leaving the afterburner expand in the nozzle from the state (08*A*) to state (9). As usual a check for nozzle choking is performed as follows:

$$\frac{P_{08A}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}} \tag{6.39}$$

If the nozzle is unchoked, the jet speed will be:

$$V_{9ab} = \sqrt{2Cp_h\eta_n T_{08A}\left[1 - (P_a/P_{08})^{\frac{\gamma_h - 1}{\gamma_h}}\right]} \tag{6.40a}$$

If the nozzle is choked, then the exhaust gases leave the nozzle with a temperature of T_{9A} which is calculated from the relation:

$$\left(\frac{T_{08}}{T_{9A}}\right) = \left(\frac{\gamma_h + 1}{2}\right)$$

The exhaust speed is then calculated from the relation:

$$V_{9ab} = \sqrt{\gamma_h R T_{9A}} \tag{6.40b}$$

6.2.8 Performance Parameters of Double-Spool Turbojet Engine

By performance parameters, it is meant the specific thrust and the specific fuel consumption as well as the three efficiencies.

The specific thrust is expressed by the relation:

$$\frac{T}{\dot{m}_a} = [(1 + f + f_{ab})V_9 - V] + \frac{A_9}{\dot{m}_a}(P_9 - P_a) \tag{6.41}$$

The thrust specific fuel consumption (*TSFC*) is given by:

$$TSFC = \frac{\dot{m}_f + \dot{m}_{fab}}{T}$$

Substituting from Eq. (6.41), the thrust specific fuel consumption is:

$$TSFC = \frac{f + f_{ab}}{(1 + f + f_{ab})V_9 - V + \frac{A_9}{\dot{m}_a}(P_9 - P_a)} \tag{6.42}$$

For inoperative afterburner, the same Eqs. (6.41) and (6.42) are used, but the afterburner fuel-to-air ratio f_{ab} is set equal to zero.

Turbojet engines resemble a one stream flow engine, and then from Chap. 2, the efficiencies are calculated as follows. The propulsive efficiency is obtained from the relation:

$$\eta_p = \frac{TV}{TV + \frac{1}{2}\dot{m}_e(V_e - V)^2}$$

where the mass and velocity of the gases leaving the nozzle are expressed in general as $\dot{m}_e$ and V_e.

The thermal efficiency is expressed as:

$$\eta_{th} = TV + \frac{\frac{1}{2}\dot{m}_e(V_e - V)^2}{\dot{m}_f Q_R}$$

The overall efficiency is then:

$$\eta_0 = \eta_p \times \eta_{th}$$

Example 6.6 The following data presents some details of a double-spool turbojet engine close to Pratt & Whitney J57:

$$\pi_{c1} = 4.0, \quad \pi_{c2} = 3.125, \quad T_{0\ \max} = 1143\ \text{K}, \quad Q_R = 45\ \text{MJ/kg}, \quad T_a = 288\ \text{K}$$
$$P_a = 101\ \text{kPa}, \quad M = 0.0, \quad \eta_{c1} = \eta_{c2} = 0.86, \quad \eta_{t1} = \eta_{t2} = 0.9, \quad \eta_b = 0.99$$
$$\eta_n = 0.94, \quad \Delta P_{cc} = 1\%, \quad \gamma_c = 1.4, \quad \gamma_h = \frac{4}{3}, \quad \text{Nozzle exhaust area } A_n = 0.208\ \text{m}^2$$

Afterburner is assumed inoperative with no pressure losses in tailpipe.

Calculate

Air mass flow rate
Thrust force
Thrust specific fuel consumption (*TSFC*)
Propulsive efficiency
Thermal efficiency
Overall efficiency

Solution
Since Mach number is zero, then the above case represents a ground run. Thus:

Intake

$$T_{02} = T_{01} = T_{0a} = T_a = 288 \text{ K}$$
$$P_{01} = P_{0a} = P_a = 101 \text{ kPa}$$

LPC

$$P_{03} = P_{02} \times \pi_{c1} = 4X101 = 404 \text{ kPa}$$
$$T_{03} = T_{02}\left(1 + \frac{\pi_{c1}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{c1}}\right) = 288\left(1 + \frac{4^{0.286} - 1}{0.86}\right) = 451 \text{ K}$$

HPC

$$P_{04} = P_{03} \times \pi_{c2} = 3.125 \times 404 = 1262.5 \text{ kPa}$$
$$T_{04} = T_{03}\left(1 + \frac{\pi_{c2}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{c2}}\right) = 451\left(1 + \frac{3.125^{0.286} - 1}{0.86}\right) = 653 \text{ K}$$

Combustion chamber

$$P_{05} = P_{04}(1 - \Delta P_{cc}\%) = 1262.5 \times 0.99 = 1250 \text{ kPa}$$
$$f = \frac{(Cp_h/Cp_c)(T_{05}/T_{04}) - 1}{\eta_b(Q_R/Cp_c T_{04}) - (Cp_h/Cp_c)(T_{05}/T_{04})}$$
$$f = \frac{(1.148/1.005)(1143/653) - 1}{0.99 \times 45{,}000/(1.005 \times 653) - (1.148/1.005)(1143/653)}$$
$$= \frac{0.999}{65.885} = 0.015169$$

HPT

Assume $\lambda = 1$, then:

$$\left(\frac{T_{06}}{T_{05}}\right) = 1 - \frac{(Cp_c/Cp_h)T_{03}}{\lambda_1(1+f)T_{05}}\left[\left(\frac{T_{04}}{T_{03}}\right) - 1\right]$$
$$= 1 - \left[\frac{(1.005/1.148)\times 451}{1.015169\times 1143}\right]\left(\frac{653}{451} - 1\right) = 0.8476$$
$$T_{06} = 969\ \text{K}$$
$$\left(\frac{P_{06}}{P_{05}}\right) = \left(1 - \frac{T_{05}-T_{06}}{\eta_{t1}T_{05}}\right)^{\frac{\gamma_t}{\gamma_t - 1}} = \left(1 - \frac{1143-969}{0.9\times 1143}\right)^4 = 0.4765$$
$$P_{06} = 595.7\ \text{kPa}$$

LPT

Assume also $\lambda_2 = 1$, then:

$$\left(\frac{T_{07}}{T_{06}}\right) = 1 - \frac{(Cp_c/Cp_h)T_{02}}{\lambda_2(1+f)T_{06}}\left[\left(\frac{T_{03}}{T_{02}}\right) - 1\right]$$
$$= 1 - \left[\frac{(1.005/1.148)\times 288}{1.015169\times 969}\right]\left(\frac{451}{288} - 1\right) = 0.8549$$
$$T_{07} = 828.4\ \text{K}$$
$$\left(\frac{P_{07}}{P_{06}}\right) = \left(1 - \frac{T_{06}-T_{07}}{\eta_{t1}T_{06}}\right)^{\frac{\gamma_t}{\gamma_t - 1}} = \left(1 - \frac{969-828.4}{0.9\times 969}\right)^4 = 0.495$$
$$P_{07} = 294.9\ \text{kPa}$$

Nozzle

No losses are assumed at tailpipe; thus, conditions at point (7) are equal to those in point (8).

Check nozzle choking:

$$\frac{P_{08}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}} = 1.9336$$
$$P_c = 152.5\ \text{kPa}$$

Since $P_c > P_a$, then the nozzle is choked and thus: $P_9 > P_c$

The exhaust gas temperature is calculated from the relation:

$$T_9 = T_c = \frac{2T_{08}}{\gamma_h + 1} = 710\ \text{K}$$

The jet speed is expressed as:

$$V_9 = \sqrt{\gamma_h R T_9} = 521.3 \ \text{m/s}$$

The density of exhaust gases: $\rho_9 = \frac{P_9}{RT_9} = 0.748 \ \text{kg/m}^3$

The gas mass flow rate: $\dot{m}_9 = \rho_9 V_9 A_9 = 0.748 \times 521.3 \times 0.208 = 81.15 \ \text{kg/s}$

$$\text{The air mass flow rate :} \qquad \dot{m}_a = \frac{\dot{m}_9}{1+f} = 79.94 \ \text{kg/s} \qquad \#$$

$$T = \dot{m}_a(1+f)V_9 + A_9(P_9 - P_a)$$
$$= 81.15 \times 521.3 + 0.208 \times (152.5 - 101) \times 10^3$$
$$T = (42.303 + 10.712) \times 10^3 = 53{,}015\,\text{N} = 53.015 \ \text{kN}$$

The thrust specific fuel consumption is:

$$TSFC = \frac{f\dot{m}}{T} = \frac{0.015169 \times 79.94}{53{,}656} = 21.79 \times 10^{-6} \ \ \text{kg/N.s} = 21.79 \ \text{g/N.s} \qquad \#$$

The propulsive efficiency is *zero* as this case represents a ground run ($V = 0.0$).

The thermal efficiency with $V = 0.0$ is expressed as:

$$\eta_{th} = \frac{\dot{m}_e(V_e)^2}{2\dot{m}_f Q_R} = \frac{(1+f)V_e^2}{2fQ_R} = 0.202 = 20.2\%$$

The overall efficiency is also zero as it is the product of both propulsive efficiency (here zero) and thermal efficiency.

Example 6.7 If the afterburner for the two-spool engine described in Example (6.6) is operative. Additional data are:

$$T_{0max} = 1850 \ \text{K}, \qquad \eta_{ab} = 98\%, \qquad \Delta \text{P}_{ab} = 2\%$$

Calculate

The new values for thrust force and *TSFC*
The thermal efficiency
The percentage increase in thrust force
The percentage increase in fuel consumption

Solutions

All the calculations in Example (6.6) are applied to the present case of operative afterburner up to the states (8 and 9).

Afterburner

$$f_{\mathrm{ab}} = \frac{(1+f)(Cp_{8\mathrm{A}}T_{08\mathrm{A}}\text{-}Cp_7T_{07})}{\eta_{ab}Q_R - Cp_{8A}T_{08A}}$$

$$f_{ab} = \frac{(1+0.015169) \times 1.148 \times (1850 - 828.4)}{0.98 \times 45{,}000 - 1.148 \times 2000} = \frac{1{,}191}{41{,}804} = 0.0285$$

$$P_{08A} = P_{07}(1 - \Delta P_{ab}\%)$$

$$P_{08A} = 289 \text{ kPa}$$

Nozzle

As usual a check for nozzle choking is performed as follows:

$$\frac{P_{08A}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}} = 1.9336$$

Then the critical pressure is $P_c = 149.46 \text{ kPa} > P_a$.

Then the nozzle is also choked. The exhaust gases leave the nozzle with a temperature of T_{9A} calculated as:

$$\left(\frac{T_{08}}{T_{9A}}\right) = \left(\frac{\gamma_h + 1}{2}\right) = 1.167$$

The exhaust temperature is then $T_{9A} = 1585$ K, and the exhaust jet speed is calculated from the relation:

$$V_{9ab} = \sqrt{\gamma_h R T_{9A}} = 779 \text{ m/s}$$

The *thrust* is expressed by the relation:

$$T_{ab} = \dot{m}_a[(1+f+f_{ab})V_9] + A_9(P_9 - P_a) = 64{,}993 + 10{,}080 = 75{,}073 \text{ N}$$
$$= 75.073 \text{ kN}$$

The *thrust specific fuel consumption* (*TSFC*) is given by:

$$TSFC = \frac{\dot{m}_f + \dot{m}_{fab}}{T} = \frac{\dot{m}_a(f + f_{ab})}{T} = 49.82 \text{ g/(kN.s)}$$

Thermal efficiency

$$\eta_{th} = \frac{\dot{m}_e(V_e)^2}{2\dot{m}_f Q_R} = \frac{(1+f+f_{ab})V_e^2}{2(f+f_{ab})Q_R} = 0.161 = 16.1 \text{ \%}$$

Increase in thrust: $\Delta T\%$

$$\Delta T\% = \frac{T_{ab} - T}{T} = \frac{76.045 - 53.656}{53.656} = 41.6\ \%$$

Increase in fuel consumption ($\Delta f\%$)

$$\Delta f\% = \frac{f_{ab} - f}{f} = \frac{0.0285 - 0.015169}{0.015169} = 87.88\%$$

6.2.9 Micro-turbojet

Small turbojet engine are developed for use in cruise missiles, target drones, and other small unmanned air vehicles (UAVs). Examples are Microturbo TRI 60 which different types (TRI 60-1, 60-2, 60-3, 60-5, 60-20, and 60-30) produce a thrust force ranging from 3.5 to 5.3 kN. This engine is a single spool, having a l*ength of* 851 mm (33.5 in), d*iameter of* 348 mm (13.7 in), and d*ry weight of* 61.2 kg (135 lb). March 20, 1983, represents the first-ever flight of a micro-turbojet-powered model aircraft, which had a three-minute flight. This engine measured 4¾″ in diameter and 13½″ long and weighed 3¾ pounds. At 85,000 rpm it produced over 9 lb of thrust and had a top speed of 97,000 rpm. Twenty years later, commercial engines of roughly similar dimensions and weight are in the 30-lb thrust class when running at 120,000 rpm. The airframe, with its twin-boom high-tail configuration, is remarkably, perhaps inevitably, similar to many of today's trainer aircraft.

The first commercial micro-turbines were made available by JPX in the early 1990s [6]. Shortly after, Germany and the Netherlands sought out to develop a powerful, lightweight, liquid-fueled micro-turbine. AMT and Schreckling Designs were the first two companies to accomplish these goals. Figure 6.9 shows a cross section of micro-turbojet engine (SR-30). Table 6.1 shows some data for micro-turbojet engines.

Figure 6.10 illustrates Harpoon missile which is an air-, surface-, or submarine-launched anti-surface (antiship) missile. It is powered by Teledyne CAE J402 small *turbojet* engine (together with a solid-propellant booster). Teledyne CAE J402 turbojet engine is thoroughly discussed in Example (6.8).

Example 6.8 Harpoon Block II (Fig. 6.10) is the world's premier antiship missile. It uses solid-propellant booster for launch and Teledyne CAE J402 turbojet for cruise. General characteristics of Teledyne turbojet engine:

Inlet diameter: 14.35 cm

Compressor type: single-stage axial compressor and single-stage centrifugal compressor, having overall pressure ratio: 5.6:1

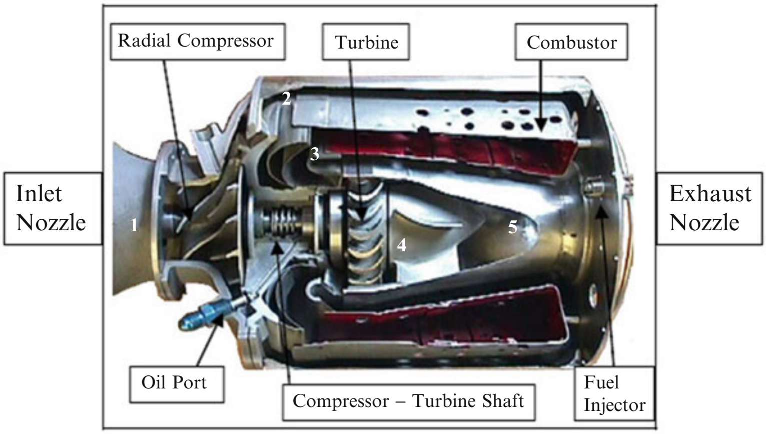


Fig. 6.9 Cross section of micro-turbojet engine (SR-30)

Table 6.1 Some micro-turbojet engines

Manufacturer	Model	Diameter (in)	Length (in)	Weight (Lbs)	Max thrust (Lbf)
AMT	Mercury HP	3.9	8.7	3.1	19.8
SWB	SWB-11 Mamba	3.5	7.3	1.9	11.4
AMT	Pegasus HP	4.7	10.4	5.9	35.3
Jetcat	P160	4.37	12.0	3.4	34
Ram	750 F	4.37	9.4	2.4	16.9
Simjet	85 N+	4.25	9.5	2.4	19.1
Artes/Espiell	JG100	4.33	9.4	2.3	22.5
Kamps		4.33	9.3	2.7	13.5
Phoenix	30/3	4.33	9.8	3.5	20.23
Schreckling	FD 3/64	4.33	NA	1.9	4.95
AMT	Olympus	5.1	10.6	5.3	42.7

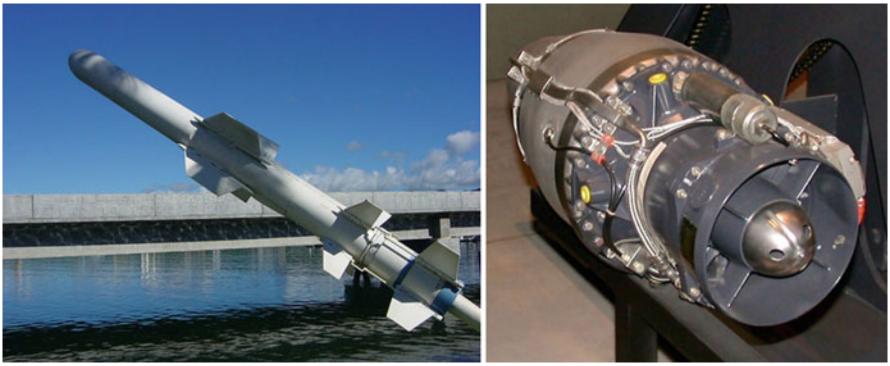


Fig. 6.10 Harpoon missile

Maximum thrust: 2.9 kN
Thrust specific fuel consumption: 0.1226 kg/N.hr

When the turbojet is turned on, the missile has a speed of 135 m/s and sea-level conditions (temperature and pressure 288 K and 101 kPa). Assume all processes are ideal as well as constant specific heat and specific heat ratio inside the engine.

Calculate

Fuel-to-air ratio
Exhaust jet speed
The maximum temperature in engine
Fuel heating value

Solution
Inlet area:

$$A_i = \frac{\pi}{4} D^2 = 0.01617 \ \text{m}^2$$

$$\dot{m}_a = \rho_a V_f A_i = 1.222 \times 250 \times 0.01617 = 4.94 \ \text{kg/s}$$

$$\dot{m}_f = TSFC \times T = \frac{0.1226}{3600} \times 2900 = 0.0988 \ \text{kg/s}$$

Fuel-to-air ratio

$$f = \frac{\dot{m}_f}{\dot{m}_a} = 0.02$$

Exhaust jet speed

From the thrust force: $T = \dot{m}_a \left[(1+f) V_e - V_f \right]$
Exhaust jet speed is:

$$V_e = \frac{1}{1+f} \left(\frac{T}{\dot{m}_a} + V_f \right) = \frac{1}{1.02} \left(\frac{2900}{4.94} + 250 \right) = 820.6 \ \text{m/s}$$

Maximum cycle temperature

The following cycle analysis is performed first:
Sonic speed at sea level:

$$a = \sqrt{\gamma R T} = 340 \ \text{m/s}$$

Flight Mach number:

$$M = \frac{V}{a} = 0.397$$

Compressor inlet (state 2):

$$T_{02} = T_a\left(1 + \frac{\gamma - 1}{2}M^2\right) = 297 \text{ K}$$

$$P_{02} = P_a\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 112.6 \text{ kPa}$$

Combustion chamber inlet (state 3):

$$P_{03} = P_{02}\pi_c = 630.5 \text{ kPa} = P_{04}$$

$$T_{03} = T_{02}(\pi_c)^{\frac{\gamma-1}{\gamma}} = 486 \text{ K}$$

With constant specific heats, then balance between compressor (states 2–3) and turbine (states 4–5) yields:

$$(1+f)C_p(T_{04} - T_{05}) = C_p(T_{03} - T_{02})$$

$$T_{05} = T_{04} - \frac{(T_{03} - T_{02})}{(1+f)} = T_{04} - 186$$

$$\text{Besides :} \qquad \frac{P_{05}}{P_{04}} = \left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{(A)}$$

Nozzle (states 5–6):

$$V_e^2 = 2C_p(T_{05} - T_6) = 2C_pT_{05}\left[1 - \left(\frac{P_a}{P_{05}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{P_a}{P_{05}} = \left(1 - \frac{V_e^2}{2C_pT_{05}}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 - \frac{(820.6)^2}{2 \times 1005}\frac{1}{T_{05}}\right]^{\frac{\gamma}{\gamma-1}} = \left(1 - \frac{335}{T_{04} - 186}\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{(B)}$$

From (A) and (B), then:

$$\frac{P_a}{P_{04}} = \left[\left(\frac{T_{05}}{T_{04}}\right)\left(1 - \frac{335}{T_{04} - 186}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{P_a}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_{04}-186}{T_{04}}\right)\left(1-\frac{335}{T_{04}-186}\right)$$

$$\left(\frac{101}{630.4}\right)^{0.296} = \left(\frac{T_{04}-186}{T_{04}}\right)\left(\frac{T_{04}-521}{T_{04}-186}\right) = \frac{T_{04}-521}{T_{04}}$$

$$0.59227 = \frac{T_{04}-521}{T_{04}}$$

Maximum cycle temperature is then:

$$T_{04} = 1278 \ \text{K}$$

Fuel heating value

From balance in combustion chamber:

$$Q_R = C_p T_{04} + \frac{C_p(T_{04}-T_{03})}{f} = 1.005 \times 1278 + \frac{1.005 \times (1278-486)}{0.02}$$
$$= 41082 \ \text{kJ/kg}$$

$$Q_R = 41.082 \ \text{MJ/kg}$$

6.3 Turbofan

6.3.1 Introduction

Turbofan engines are the dominant air-breathing engines in the last 4 decades. They are the most reliable engines ever developed. Turbofans were first termed by Rolls-Royce as *bypass turbojet*. Boeing Company sometimes identifies them as *fanjets* [7]. In turbofan or "bypass" engine, the partly compressed airflow is divided, some into a central part, the gas generator or core, and some into a surrounding casing, the bypass or fan duct. The gas generator acts like a turbojet, while the bypass air is accelerated relatively slowly down the duct to provide "cold stream" thrust. The cold and hot streams mix inside or outside the engine to give better propulsive efficiency, lower noise levels, and improved fuel consumption.

Gas generator in both of turbofan and turbojet engines has three modules, namely, compressor, combustion chamber, and turbine. In turbofan engines, the fan pressurizes air and feeds it aft. Most goes around the engine core through a nozzle-shaped chamber. The rest goes through the engine core where it mixes with fuel and ignites. The hot gases expand through the turbine section and next the hot nozzle as it exits the engine.

In the 1950s, Rolls-Royce introduced the first turbofan in the world, namely, RB.80 Conway. Conway design and development started in the 1940s, but it was used only in the late 1950s and early 1960s. The Conway powered versions of the Handley Page Victor, Vickers VC10, Boeing 707-420, and Douglas DC-8-40. It had a very low-bypass ratio (BPR designated as β) of 0.25 and maximum thrust of 76.3 kN [8]. In 1964, another RR turbofan engine was produced having also a low-bypass ratio of 0.64. GE led the way of high-bypass ratio turbofan engines in 1965 with the TF39 (BPR 8) to power the C-5 Galaxy [9]. Rolls-Royce developed the first worldwide three-spool turbofan engines RB211 in 1969 having a high-bypass ratio ranging from 4.0 to 5.0 depending on its series [10].

GE still leads HBPR turbofan engines with GE90 (BPR of 9) [11]. The maximum achieved BPR is 12.0:1 which is a record number scored by Pratt and Whitney in 2013 in its PW1500 series, which powered Bombardier CSeries CS100 and CS300 (second half of 2015), and PW 1100G series, which powered Airbus A319neo, A320neo, and A321neo in October 2015 [12]. The PW1400 series will power the Russian Federation airplane Irkut MC-21 in 2017, whereas the PW1700 series will power Embraer E-Jets E2 in 2018.

There are several advantages to turbofan engines over both of turboprop and the turbojet engines. The fan is not as large as the propeller, so the increase of speeds along the blade is less. Thus, turbofan engines power now all civil transports flying at transonic speeds up to Mach 0.9. Also, by enclosing the fan inside a duct or cowling, the aerodynamics is better controlled. There is less flow separation at the higher speeds and less trouble with shock developing. The turbofan may suck in more airflow than a turbojet, thus generating more thrust. Like the turboprop engine, the turbofan has low fuel consumption compared with the turbojet. The turbofan engine is the best choice for high-speed, subsonic commercial airplanes.

6.3.2 Milestones

Appendix C presents detailed list of milestones of turbofan engines.

6.3.3 Classifications of Turbofan Engines

As described in Chap. 1, numerous types of turbofan exist. Figure 6.11 illustrates a very detailed classification of turbofan engines.

Turbofan engines may be classified based on fan location as either forward or aft fan. Based on a number of spools, it may be classified as single, double, and three (triple) spools. Based on a bypass ratio, it may be categorized as either low- or high-bypass ratio. The fan may be geared or ungeared to its driving low-pressure turbine. Moreover, mixed types (low-bypass types) may be fitted with afterburner or not. Cross matching between different categories is identified in Fig. 6.11.

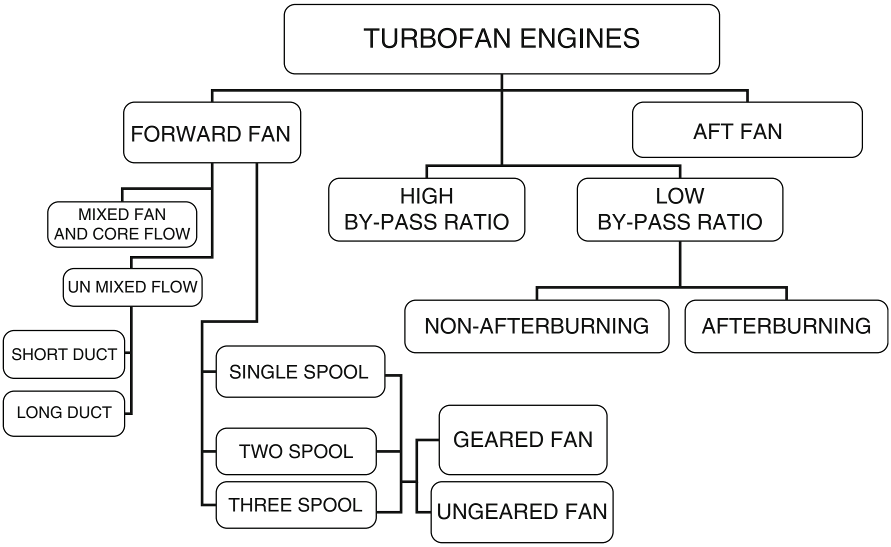


Fig. 6.11 Classification of turbofan engines

High-bypass ratio turbofan ($\beta = 7 - 8$) achieves around 75 % of its thrust from the bypass air and is ideal for subsonic transport aircraft. *Examples in commercial usage are Rolls-Royce Trent series (500/700/800/900/1000 powering in the Airbus A330, A340, A350, and A380), Pratt & Whitney PW1000 G (geared) powering Airbus A320neo, Bombardier CSeries, Embraer E2, Mitsubishi Regional Jet MC-21, and General Electric GE90 series (76B/77B/85B/90B/92B/94B/110B1/115B) powering Boeing 777-300ER, 747.*

A low-bypass ratio turbofan, where the air is divided approximately equally between the gas generator and the bypass duct, is well suited to high-speed military usage. Examples *in military usage are Rolls-Royce RB199 in the Tornado*, Pratt & Whitney *F100-PW-200 in F-16A/B and F-15, as well as* EuroJet Turbo GmbH *EJ200 powering the Typhoon*.

It is too lengthy to analyze all types of turbofan engines. So, the following cases will be analyzed:

Double-spool unmixed flow turbofan
Tripe-spool unmixed turbofan
Double-spool mixed turbofan with afterburner
Aft fan engine

First of all, let us define the bypass ratio as:

$$\beta = BPR = \frac{\dot{m}_{\text{cold}}}{\dot{m}_{\text{hot}}} \equiv \frac{\dot{m}_{\text{fan}}}{\dot{m}_{\text{core}}}$$

Table 6.2 Component efficiencies and specific heat ratio for moderate bypass ratio ($\beta = 2.7$) [13]

Module	Efficiency (η)	Specific heat ratio (γ)
Intake (diffuser)	0.97 ($M < 1$)	1.4
	0.85 ($M > 1$)	
Fan	0.85	1.39
Compressor	0.85	1.37
Turbine	0.9	1.33
Core nozzle	0.97	1.36
Fan nozzle	0.97	1.39

Thus, if the air mass through the core (compressor) is ($\dot{m}_a$), then the bypass air (fan duct) mass flow rate is ($\beta\dot{m}_a$).Typical component efficiencies and specific heat ratios for a supersonic turbofan engine having a bypass ratio, $\beta = 2.7$, are given in Table 6.2.

6.3.4 Forward Fan Unmixed Double-Spool Configuration

The main components here are the intake, fan, fan nozzle, low-pressure compressor (booster), high-pressure compressor, combustion chamber, high-pressure turbine, low-pressure turbine, and turbine nozzle. Two cases are seen for the low-pressure spool, namely:

Fan and low-pressure compressor (LPC) on one shaft and driven by low-pressure turbine

Fan driven by the LPT and the compressor driven by the HPT

Typical examples for the first type are the General Electric CF6 engine series and Pratt & Whitney PW4000 series. Typical example for the second type is GE Rolls-Royce F136 engine.

A schematic diagram for the first type as well as its T-s diagram is shown in Figs. 6.12 and 6.13.

Low-pressure spool is rotating with N_1 speed, while the high-pressure spool is rotating with N_2 speed.

Here below is an analysis for the different modules of the engine:

1. Intake

The inlet module is analyzed quite similar to turbojet engine using Eqs. (6.1), (6.2), and (6.3).

2. Fan

A similar analysis to the compressor section is followed. The appropriate equations are:

$$P_{010} = (P_{02})\left(\pi_f\right) \tag{6.43}$$

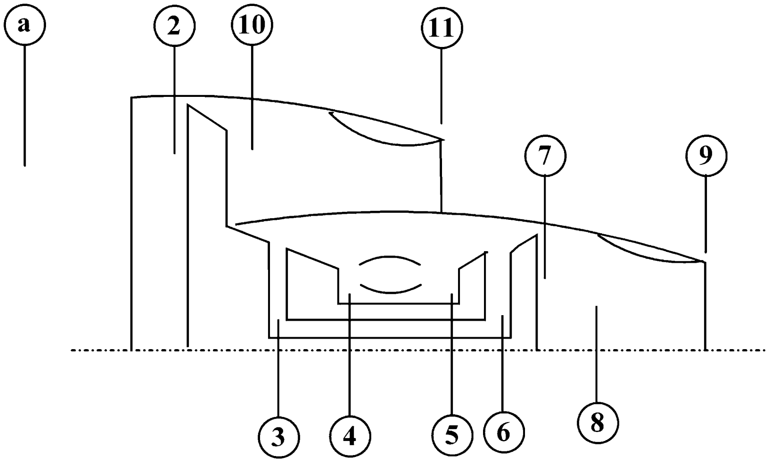


Fig. 6.12 Layout of a double spool where fan and LPC driven by LPT

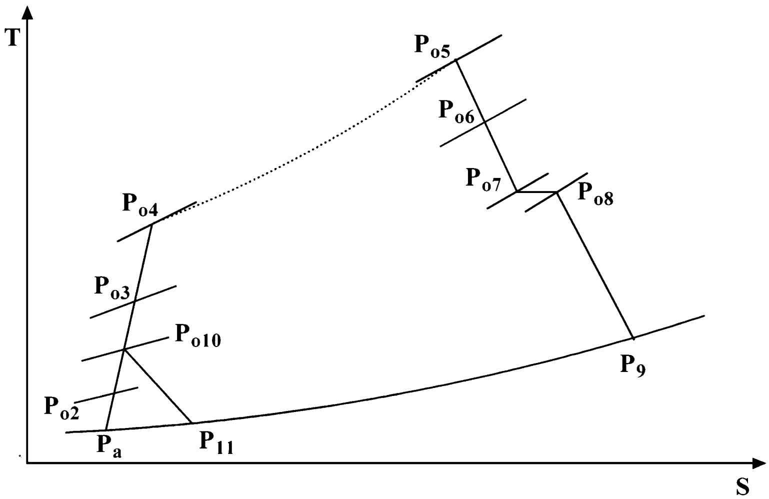


Fig. 6.13 T-s diagram for a double-spool turbofan engine

$$T_{010} = T_{02}\left[1 + \frac{\left(\pi_f^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_f}\right] \tag{6.44}$$

3. Low-pressure compressor (or booster)

Low-pressure ratio is developed in this compressor (normally less than two):

$$P_{03} = (P_{010})(\pi_{\text{lpc}}) \tag{6.45}$$

$$T_{03} = T_{010}\left[1 + \frac{\left(\pi_{\mathrm{lpc}}^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_{\mathrm{lpc}}}\right] \tag{6.46}$$

4. High-pressure compressor

High-pressure ratio is developed by HPC (normally around 10):

$$P_{04} = (P_{03})\left(\pi_{\mathrm{hpc}}\right) \tag{6.47}$$

$$T_{04} = T_{03}\left[1 + \frac{\left(\pi_{\mathrm{hpc}}^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_{\mathrm{hpc}}}\right] \tag{6.48}$$

5. Combustion chamber

The pressure at the outlet of the combustion chamber is obtained from the pressure drop in the combustion chamber:

$$P_{05} = P_{04} - \Delta P_{cc} \tag{6.49a}$$

$$\text{Or} \qquad P_{05} = P_{04}(1 - \Delta P_{cc}\%) \tag{6.49b}$$

The temperature at the outlet of the combustion chamber is also the maximum temperature in the engine and known in advance. Thus, the fuel-to-air ratio is calculated from the relation:

$$f = \frac{\left(\frac{Cp_h}{Cp_c}\right)\left(\frac{T_{05}}{T_{04}}\right) - 1}{\eta_b\left(\frac{Q_R}{Cp_c T_{04}}\right) - \left(\frac{Cp_h}{Cp_c}\right)\left(\frac{T_{05}}{T_{04}}\right)} \tag{6.50}$$

6. High-pressure turbine (HPT)

To calculate the temperature and pressure at the outlet of the high-pressure turbine, an energy balance between the high-pressure compressor (HPC) and high-pressure turbine (HPT) is performed:

$$\dot{m}_a Cp_c(T_{04} - T_{03}) = \eta_{m1}\lambda_1[\dot{m}_a(1+f)Cp_h(T_{05} - T_{06})] \tag{6.51a}$$

$$\left(\frac{T_{06}}{T_{05}}\right) = 1 - \frac{(Cp_c/Cp_h)T_{03}}{\lambda_1\eta_{m1}(1+f)T_{05}}\left[\left(\frac{T_{04}}{T_{03}}\right) - 1\right] \tag{6.51b}$$

Here also the mechanical efficiency for high-pressure spool is (η_{m1}), and the portion of energy extracted by the HPC from that developed in the HPT is (λ_1). From the above relation, the temperature at the outlet of the turbine T_{06} is calculated. Moreover, from the known isentropic efficiency of high-pressure turbine η_{hpt}, the outlet pressure from the high-pressure turbine P_{06} is calculated from the relation:

$$P_{06} = P_{05}\left(1 - \frac{T_{05} - T_{06}}{\eta_{\text{hpt}} \times T_{05}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} \tag{6.52}$$

7. Low-pressure turbine

An energy balance between the fan and low-pressure compressor (LPC) from one side and the low-pressure turbine (LPT) on the other side is expressed by the relation:

$$\begin{aligned}&\beta\dot{m}_a Cp_c(T_{010} - T_{02}) + \dot{m}_a Cp_c(T_{03} - T_{02})\\ &\quad = \eta_{m2}\lambda_2[\dot{m}_a(1+f)Cp_h(T_{06} - T_{07})]\end{aligned} \tag{6.53a}$$

or

$$\begin{aligned}&(1+\beta)\dot{m}_a Cp_c(T_{010} - T_{02}) + \dot{m}_a Cp_c(T_{03} - T_{010})\\ &\quad = \eta_{m2}\lambda_2\dot{m}_a(1+f)Cp_h(T_{06} - T_{07})\end{aligned} \tag{6.53b}$$

$$T_{07} = T_{06} - \frac{Cp_c}{\eta_{m2}\lambda_2(1+f)Cp_h}[(1+\beta)(T_{010} - T_{02}) + (T_{03} - T_{010})]$$

The pressure at the outlet is obtained from the relation:

$$P_{07} = P_{06}\left(1 - \frac{T_{06} - T_{07}}{\eta_{\text{lpt}} \times T_{06}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} \tag{6.54}$$

Bleed from high-pressure compressor

Now, if there is an *air bleed* from the high-pressure compressor (HPC) at a station where the pressure is P_{03b}, then the energy balance with the high-pressure turbine gives:

$$\begin{aligned}&\dot{m}_a Cp_c(T_{03b} - T_{03}) + \dot{m}_a(1-b)Cp_c(T_{04} - T_{03b})\\ &\quad = \eta_{m1}\lambda_1\dot{m}_a(1+f-b)Cp_h(T_{05} - T_{06})\end{aligned} \tag{6.51c}$$

where $b = \frac{\dot{m}_b}{\dot{m}_a}$ is the air bleed ratio defining the ratio between the air bled from the HPC to the core airflow rate. Moreover, such a bleed has its impact on the energy balance of the low-pressure spool as the air passing through the low-pressure turbine is now reduced:

$$\begin{aligned}&(1+\beta)\dot{m}_a Cp_c(T_{010} - T_{02}) + \dot{m}_a Cp_c(T_{03} - T_{010})\\ &\quad = \eta_{m2}\lambda_2\dot{m}_a(1+f-b)Cp_h(T_{06} - T_{07})\end{aligned} \tag{6.53c}$$

The flow in the jet pipe is frequently associated with a pressure drop mainly due to skin friction.

Thus, the pressure upstream of the turbine nozzle is slightly less than the outlet pressure from the turbine. The temperature, however, is the same. Thus:

$$P_{08} = P_{07}\left(1 - \Delta P_{\text{jet pipe}}\right)$$
$$T_{08} = T_{07}$$

8. Turbine nozzle

The exhaust velocities of both of the hot gases from the turbine nozzles are obtained after checks for choking. Thus, if the isentropic efficiency of turbine nozzle is η_{n_t}, then the critical pressure is calculated from the relation:

$$\frac{P_{08}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_{n_t}}\left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}}$$

For an ideal, $\eta_{n_t} = 1$, then the above equation is reduced to:

$$\left(\frac{P_{08}}{P_c}\right) = \left(\frac{\gamma_h + 1}{2}\right)^{\frac{\gamma_h}{\gamma_h - 1}}$$

If $P_c \geq P_a$ then the nozzle is choked. The temperature and pressure of the gases leaving the nozzle are the critical values ($T_9 = T_c, P_9 = P_c$). Temperature is obtained from the relation:

$$\left(\frac{T_{08}}{T_9}\right) = \left(\frac{\gamma_h + 1}{2}\right)$$

In this case the gases leave the nozzle at a speed equal to the sonic speed or:

$$V_9 = \sqrt{\gamma_h R T_9} \tag{6.55a}$$

If the nozzle is unchoked ($P_9 = P_a$), then the speed of the gases leaving the nozzle is now given by:

$$V_9 = \sqrt{2Cp_h T_{08} \eta_{nt}\left[1 - \left(P_a/P_{08}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]} \tag{6.55b}$$

The pressure ratio in the nozzle is obtained from the relation:

$$\frac{P_{08}}{P_a} = \frac{P_{08}}{P_{07}} \frac{P_{07}}{P_{06}} \frac{P_{06}}{P_{05}} \frac{P_{05}}{P_{04}} \frac{P_{04}}{P_{03}} \frac{P_{03}}{P_{010}} \frac{P_{010}}{P_{02}} \frac{P_{02}}{P_{0a}} \frac{P_{0a}}{P_a}$$

9. Fan nozzle

The critical pressure is calculated from the relation:

$$\frac{P_{010}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_{fn}}\left(\frac{\gamma_c - 1}{\gamma_c + 1}\right)\right]^{\frac{\gamma_c}{\gamma_c - 1}}}$$

Also for an ideal nozzle $\eta_{fn} = 1$ and the above equation will be reduced to:

$$\left(\frac{P_{010}}{P_c}\right) = \left(\frac{\gamma_c + 1}{2}\right)^{\frac{\gamma_c}{\gamma_c - 1}}$$

If $P_c \geq P_a$ then the fan nozzle is choked. The temperature and pressure of air leaving the nozzle are ($T_{11} = T_c$, $P_{11} = P_c$). Temperature is then obtained from the relation:

$$\left(\frac{T_{010}}{T_{11}}\right) = \left(\frac{\gamma_c + 1}{2}\right)$$

Then the gases leave the nozzle at a speed equal to the sonic speed or:

$$V_{11} = \sqrt{\gamma_c R T_{11}} \tag{6.56a}$$

If the nozzle is unchoked ($P_{11} = P_a$), then the speed of the gases leaving the nozzle is now given by:

$$V_{11} = \sqrt{2C_{pc}T_{010}\eta_{fn}\left[1 - (P_a/P_{010})^{\frac{\gamma_c - 1}{\gamma_c}}\right]} \tag{6.56b}$$

$$\frac{P_{010}}{P_a} = \frac{P_{010}}{P_{02}}\frac{P_{02}}{P_a}$$

The thrust force is now obtained from the general relation:

$$\frac{T}{\dot{m}_a} = (1+f)V_9 + \beta V_{11} - U(1+\beta) + \frac{1}{\dot{m}_a}\left[A_{11}(P_{11} - P_a) + A_9(P_9 - P_a)\right]$$

$$= (1+f)V_9 + \beta(V_{11} - U) - U + \frac{1}{\dot{m}_a}\left[A_{11}(P_{11} - P_a) + A_9(P_9 - P_a)\right] \tag{6.57a}$$

The specific thrust, the thrust force per total air mass flow rate ($\dot{M}_{at}$), is further evaluated from the relation:

$$\frac{T}{\dot{M}_{at}} = \frac{T}{\dot{m}_h + \dot{m}_c} = \frac{T}{\dot{m}_a(1+\beta)} = \frac{(1+f)}{(1+\beta)}V_9 + \frac{\beta}{(1+\beta)}V_{11} - U + \frac{1}{\dot{m}_a(1+\beta)}[A_{11}(P_{11} - P_a) + A_9(P_9 - P_a)] \tag{6.57b}$$

The thrust specific fuel consumption is:

$$TSFC = \dot{m}_f/T = \frac{\dot{m}_f}{\dot{m}_a}\frac{\dot{m}_a}{T} = \frac{f}{T/\dot{m}_a}$$

It is noted from the above equation that the fuel-to-air ratio is calculated as the ratio between fuel flow rate and the airflow through the core of engine and not the total airflow rate through the engine.

The thrust force, specific thrust, and the thrust specific fuel consumption are calculated.

Example 6.9 The Tomahawk is a long-range subsonic cruise missile powered by a solid-fuel rocket booster and the small two-spool turbofan engine Williams International F107-WR-402 (Fig. 6.14). It has the following data:

Flight speed $V_f = 247.22$ m/s
Ambient temperature $T_a = 275$ K
Ambient pressure $P_a = 0.79$ bar
Thrust force $T = 3.1$ kN
Specific fuel consumption = 0.682 kg/kg-h
Bypass ratio = 1
Overall pressure ratio = 13.8
Fan pressure ratio = 2.1
Fan diameter = 0.305 m
Fuel heating value = 43,000 kJ/kg

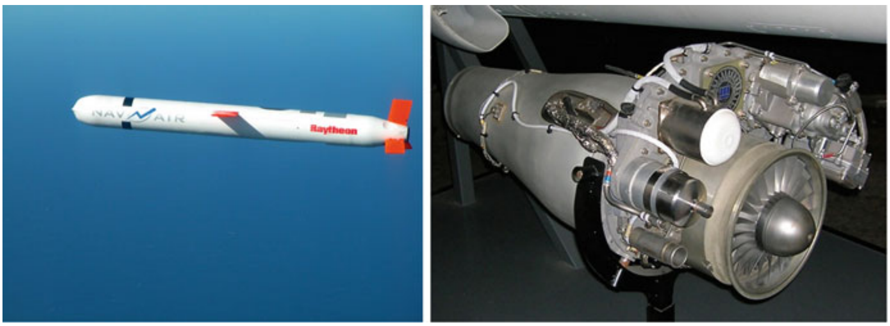
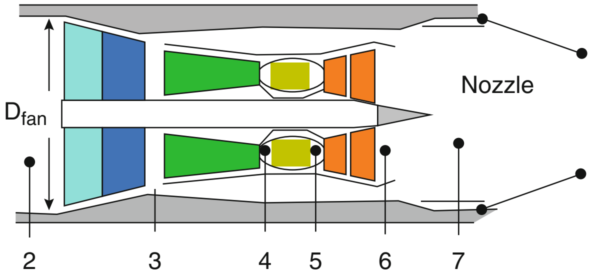


Fig. 6.14a Tomahawk missile and Williams F107-WR-402 turbofan engine

Fig. 6.14b Schematic diagram of Williams F107-WR-402 turbofan engine



Calculate

Air mass flow rate
Fuel-to-air ratio
Exhaust gas speed
Engine maximum temperature

Solution

Air mass flow rate

Flight speed

$$V_f = 247.22 \text{ m/s}$$

$$A = 0.07306 \text{ m}^2$$

$$\rho_a = \frac{P_a}{RT_a} = 1.001 \text{ kg/m}^3$$

$$\dot{m}_a = \rho_a V_f A = 18.08 \text{ kg/s}$$

$$\text{Since} \quad \beta = 1.0, \quad \text{then} \quad \dot{m}_c = \dot{m}_h = 9.04 \text{ kg/s}$$

$$T/\dot{m}_a = \frac{T}{\dot{m}_a} = 171.46 \text{ m/s}$$

$$TSFC = 0.682 \text{ kg/kgf.h} = \frac{0.682}{9.81 \times 3600} = 0.0000193 \text{ kg/N.s} = 19.3 \text{ g/kN.s}$$

Fuel-to-air ratio

$$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_f/\dot{m}_h}{T/\dot{m}_h} = \frac{(\dot{m}_f/\dot{m}_h)}{(T/\dot{m}_a) \times (1+\beta)} = \frac{f}{(T/\dot{m}_a) \times (1+\beta)}$$

$$f = TSFC \times \left(\frac{T}{\dot{m}_a}\right)(1+\beta) = 0.0000193 \times 171.46 \times 2 = 0.006618$$

Exhaust gas speed

$$T = \left[\dot{m}_c + {}^c\dot{m}_h(1+f)^h\right]V_e - \dot{m}_a V_f$$

$$\frac{T}{\dot{m}_a} = \left[\frac{\beta}{1+\beta} + \frac{(1+f)}{1+\beta}\right]V_e - V_f = \left(\frac{\beta+1+f}{1+\beta}\right)V_e - V_f$$

$$V_e = \frac{(T/\dot{m}_a) + V_f}{\left(\frac{\beta+1+f}{1+\beta}\right)} = \frac{171.4622 + 247.22}{\left(\frac{2.006618}{2}\right)} = 417.3 \text{ m/s}$$

Engine maximum temperature is analyzed from cycle analysis.

Intake

$$M = \frac{V_f}{\sqrt{\gamma R T_a}} = 0.7437$$

$$P_{02} = P_a\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 114.0522 \text{ kPa}$$

$$T_{02} = T_a\left(1 + \frac{\gamma-1}{2}M^2\right) = 305.42 \text{ K}$$

Fan

$$P_{03} = P_{02}\pi_f = 239.51 \text{ kPa}$$

$$T_{03} = T_{02}(\pi_f)^{\frac{\gamma-1}{\gamma}} = 377.62 \text{ K}$$

Compressor

$$P_{04} = P_{03}\pi_c = 1574 \text{ kPa}$$

$$T_{04} = T_{03}(\pi_c)^{\frac{\gamma-1}{\gamma}} = 645 \text{ K}$$

Combustion chamber

$$\dot{m}_f Q_R + \dot{m}_h C_{pc} T_{04} = (\dot{m}_h + \dot{m}_f) C_{ph} T_{05}$$

$$T_{05} = \frac{f Q_R + C_{pc} T_{04}}{(1+f) C_{ph}} = \frac{0.006618 \times 43000 + 1.005 \times 645.0}{1.006618 \times 1.148} = 807.2 \text{ K}$$

Maximum temperature is then $T_{0\max} = 807.2$ K.

Example 6.10 A two-spool forward unmixed turbofan engine is powering an airliner flying at Mach number 0.9 at an altitude of 10,500 m (ambient pressure,

Table 6.3 Modules results

Module	Fan	LPC	HPC	HPT	LPT
ΔT_0	42	43	355	380	390
π	1.58	1.5	9.75	2.75	3.9

temperature, sonic speed, and air density ratio are 24,475 Pa, 220 K, 297.3 m/s, and 0.3165, respectively). The low-pressure spool is composed of a turbine driving the fan and the low-pressure compressor. The high-pressure spool is composed of a high-pressure compressor and a high-pressure turbine. Air is bled from the outlet of high-pressure compressor. The total temperature difference (in K) and pressure ratios for different modules are recorded and shown in Table 6.3:

Diffuser, fan, and turbine nozzles have isentropic efficiency of 0.9. Maximum cycle temperature is 1500 K. Inlet area is 3.14 m^2. Assume the following values for the different variables:

$\eta_b = 0.96$, $Q_{HV} = 45{,}000\ \text{kJ/kg}$, $\gamma_{\text{air}} = 1.4$, and $\gamma_{\text{gases}} = 1.33$

It is required to calculate:

(a) Pressure recovery of diffuser r_d
(b) Total air mass flow rate
(c) Isentropic efficiency of fan and low-pressure and high-pressure compressors
(d) The fuel-to-air ratio (f)
(e) The air bleed ratio (b)
(f) The bypass ratio (β)
(g) Area of cold and hot nozzles
(h) The thrust force

Solution

The modules of the engine and its different processes plotted on the T-s diagram are shown in Figs. 6.12 and 6.13.

Diffuser

$$T_{02} = T_{0a} = T_a\left(1 + \frac{\gamma_c - 1}{2} M_a^2\right) = 220 \times (1 + 0.2 \times 0.81) = 255.6\ \text{K}$$

$$P_{0a} = P_a\left(1 + \frac{\gamma_c - 1}{2} M_a^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} = (1 + 0.2 \times 0.81)^{3.5} = 1.69\ P_a$$

$$P_{02} = P_a\left(1 + \eta_d \frac{\gamma_c - 1}{2} M_a^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} = (1 + 0.9 \times 0.2 \times 0.81)^{3.5} = 1.61\ P_a$$

$$P_{02} = 39.4\ \text{kPa}$$

$$\pi_d = \frac{P_{02}}{P_{0a}} = 0.953$$

$$\rho_a = 0.3165 \rho_{s.l} = 0.3877\ \text{kg/m}^3$$

$$V = Ma = 0.9 \times 297.3 = 267.6 \text{ m/s}$$
$$\dot{m}_{\text{total}} = \rho_a V A_{inlet} = 0.3877 \times 267.6 \times 3.14 = 325.7 \text{ kg/s}$$

Fan

$$T_{02} = 255.6 \text{ K}$$
$$T_{010} = T_{02} + \Delta T_{0f} = 255.6 + 42 = 297.6 \text{ K}$$
$$P_{010} = P_{02} \times \pi_f = 39.4 \times 1.58 = 62.27 \text{ kPa}$$

The fan isentropic efficiency may be expressed as:

$$\eta_f = \frac{T_{010s} - T_{02}}{T_{010} - T_{02}} = \frac{T_{02}\left(\pi_f^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)}{\Delta T_{0f}} = \frac{255.6}{42}\left(1.58^{0.286} - 1\right) = 0.8505 = 85.05 \ \%$$

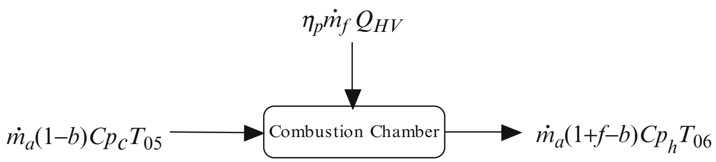
Low-pressure compressor

$$T_{03} = T_{010} + \Delta T_{0\text{LPC}} = 297.6 + 43 = 340.6 \text{ K}$$
$$P_{03} = P_{010} \times \pi_{\text{LPC}} = 62.27 \times 1.5 = 93.04 \text{ kPa}$$
$$\eta_{\text{LPC}} = \frac{T_{010}\left(\pi_{\text{LPC}}^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)}{\Delta T_{0\text{LPC}}} = \frac{297.6}{43}\left(1.5^{0.286} - 1\right) = 0.85095 = 85.1\%$$

High-pressure compressor

$$T_{04} = T_{03} + \Delta T_{0\text{HPC}} = 340.6 + 355 = 695.6 \text{ K}$$
$$P_{04} = P_{03} \times \pi_{\text{HPC}} = 93.40 \times 9.75 = 910.65 \text{ kPa}$$
$$\eta_{\text{HPC}} = \frac{T_{03}\left(\pi_{\text{HPC}}^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)}{\Delta T_{0\text{HPC}}} = \frac{340.6}{355}\left(9.75^{0.286} - 1\right) = 0.88079 = 88.1\%$$

Combustion chamber



Fuel-to-air ratio

$$f = \frac{\dot{m}_f}{\dot{m}_h} = \frac{(1-b)(Cp_h T_{05} - Cp_c T_{04})}{\eta_{cc} Q_{HV} - Cp_h T_{05}} = \frac{(1-b)(1.148 \times 1500 - 1.005 \times 695.6)}{0.96 \times 45{,}000 - 1.148 \times 1500}$$
$$= 0.02466(1-b)$$

$$f = 0.02466(1-b) \qquad (1)$$

Energy balance for high-pressure spool

$$Cp_c(T_{04} - T_{03}) = (1 + f - b)Cp_h(T_{05} - T_{06})$$

Substitute from (1)

$$1.005 \times (355) = [1 + 0.02466 \times (1-b) - b] \times 1.148 \times (380)$$
$$b = 20.2\%$$

Bleed = 20.2 %

Comment: Though this bleed ratio looks like a great percentage, however, it will be described in the turbine cooling sections that each blade row may need some 2 % of air mass flow rate for cooling, and the two turbines here have a total of five stages and so ten blade rows. In addition bleeding has its other applications for cabin air conditioning as well as other anti-icing applications.

From equation (1)

Fuel-to-air ratio $f = 0.01968$

This fuel-to-air ratio is a reasonable figure which assures the value previously obtained for the bleed ratio.

Energy balance for low-pressure spool:

$$(1+\beta)Cp_c \Delta T_{0f} + Cp_c \Delta T_{0\text{LPC}} = (1 + f - b)Cp_h T_{0\text{LPT}}$$
$$(1+\beta) \times 1.005 \times 42 + 1.005 \times 43 = (1 + 0.01968 - 0.202) \times 1.148 \times 390$$
$$\beta = 6.649$$

Thus, the *bypass ratio* is $\beta = 6.649$.

Fan airflow rate is then $\dot{m}_{\text{fan}} = \frac{\beta}{1+\beta}\dot{m}_{\text{total}} = \frac{6.649}{7.649} \times 325.7 = 283.12 \text{ kg/s}$.

Core airflow rate is then $\dot{m}_{\text{core}} = \frac{1}{1+\beta}\dot{m}_{\text{total}} = \frac{1}{7.649} \times 325.7 = 42.58 \text{ kg/s}$.

Fan nozzle

The first step in nozzle analysis is to check whether it is choked or not by calculating the pressure ratio:

$$\frac{P_{010}}{P_c} = \frac{1}{\left(1 - \frac{1}{\eta_{fn}}\frac{\gamma_c - 1}{\gamma_c + 1}\right)^{\frac{\gamma_c}{\gamma_c - 1}}} = \frac{1}{\left(1 - \frac{1}{0.9}\frac{1.4-1}{1.4+1}\right)^{\frac{1.4}{1.4-1}}} = 2.0478$$

$$\frac{P_{010}}{P_a} = \frac{62.27}{24.47} = 2.5447$$

Since $\frac{P_{010}}{P_c} < \frac{P_{010}}{P_a}$, then the nozzle is *choked*:

$$P_{11} = P_c = 30.41 \text{ kPa}$$

$$T_{11} = T_c = \frac{2}{\gamma_c + 1} T_{010} = \frac{297.6}{1.2} = 248 \text{ K}$$

$$V_{11} = V_{e\,\text{fan}} = \sqrt{\gamma_c R T_{11}} = \frac{297.6}{1.2} = 315.67 \text{ m/s}$$

$$\rho_{11} = \frac{P_{11}}{R T_{11}} = 0.4273 \text{ kg/m}^3$$

$$A_{11} = \frac{\dot{m}_{\text{fan}}}{\rho_{11} V_{11}} = 2.099 \text{ m}^2$$

Turbine nozzle

Here also a check for the choking of turbine nozzle is followed:

$$\frac{P_{08}}{P_c} = \frac{1}{\left(1 - \frac{1}{\eta_{tn}}\frac{\gamma_h - 1}{\gamma_h + 1}\right)^{\frac{\gamma_h}{\gamma h_c - 1}}} = \frac{1}{\left(1 - \frac{1}{0.9}\frac{1.333-1}{1.333+1}\right)^{\frac{1.333}{1.333-1}}} = 1.9835$$

$$\frac{P_{08}}{P_a} = \frac{84.91}{24.47} = 3.469$$

Since $\frac{P_{08}}{P_c} < \frac{P_{08}}{P_a}$, then the nozzle is *choked*:

$$P_9 = P_c = 42.81 \text{ kPa}$$

$$T_9 = T_c = \frac{2}{\gamma_h + 1} T_{08} = \frac{730}{1.167} = 625.53 \text{ K}$$

$$V_9 = V_{e\,h} = \sqrt{\gamma_h R T_9} = 489.26 \text{ m/s}$$

$$\rho_9 = \frac{P_9}{R T_9} = 0.23845 \text{ kg/m}^3$$

$$A_9 = \frac{\dot{m}_{\text{core}}(1 + f - b)}{\rho_9 V_9} = 0.298 \ m^2$$

Thrust force (T)

$$T = \dot{m}_{\text{fan}} V_{e_{\text{Fan}}} + (1 + f - b)\dot{m}_h V_{e\,h} - \dot{m}_{\text{total}} V_{\text{flight}} + P_{11} A_{11} + P_9 A_9 - P_a A_i$$
$$T = 283.12 \times 315.67 + 0.81768 \times 42.58 \times 489.26 - 325.7 \times 267.6$$
$$+10^3 (30.4 \times 2.099 + 42.81 \times 0.298 - 24.475 \times 3.14)$$
$$\therefore T = 18.97 \text{ kN}$$

6.3.5 *Forward Fan Mixed-Flow Engine*

Mixed turbofan engines are always found in either single- or two-spool engines. It was used in the past for military applications only. Nowadays it is used in many civil aircrafts but nearly all military aircrafts. An example for a two-spool engine in civil aircrafts is the CFM56 series. The cold compressed air leaving the fan will not be directly exhausted as previously described, but it flows in a long duct surrounding the engine core and then mixes with the hot gases leaving the low-pressure turbine. Thus, the cold air is heated while the hot gases are cooled. Only one mixed exhaust is found.

6.3.5.1 Mixed-Flow Two-Spool Engine

Most of the mixed turbofan engines now are two-spool ones. If mixed turbofan engines are analyzed versus unmixed turbofan engines, reasonable improvements [14] in the following points are noticed; thrust generated, noise reduction and reverse thrust increase.

Hereafter, a detailed analysis of this category will be given. Figures 6.15 and 6.16 present the engine layout and its temperature–entropy diagram.

The requirements for the mixing process are equal static pressures and also equal velocities. Thus, from the layout designation, these two conditions are specified as $P_3' = P_7$ and $V_3' = V_7$ which means that if no pressure losses in the bypass duct connecting the cold and hot streams and no pressure loss in the mixing process, then:

$$P_{010} = P_{03}' = P_{07} = P_{08} \tag{6.58}$$

If losses exist in the fan bypass duct, then:

$$P_{03}' = P_{010} - \Delta P_{\text{fan duct}} \quad \text{and} \quad P_{07} = P_{08} = P_{03}'$$

In the above equation, no pressure drop is considered during the mixing process.

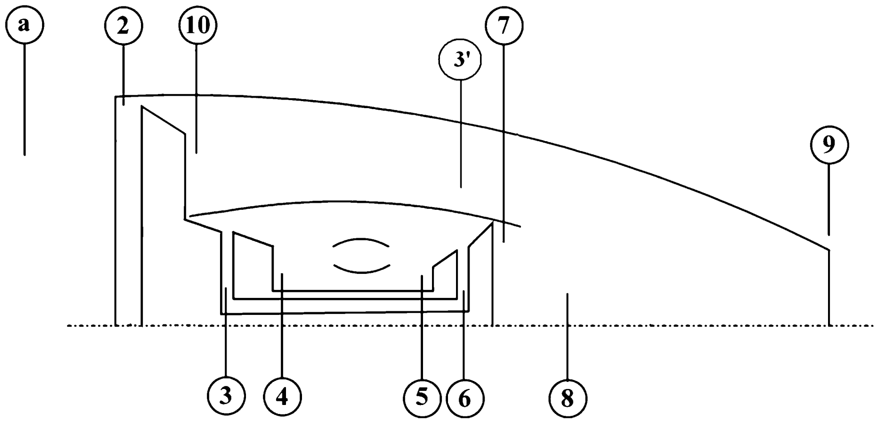


Fig. 6.15 Layout of a mixed two-spool turbofan

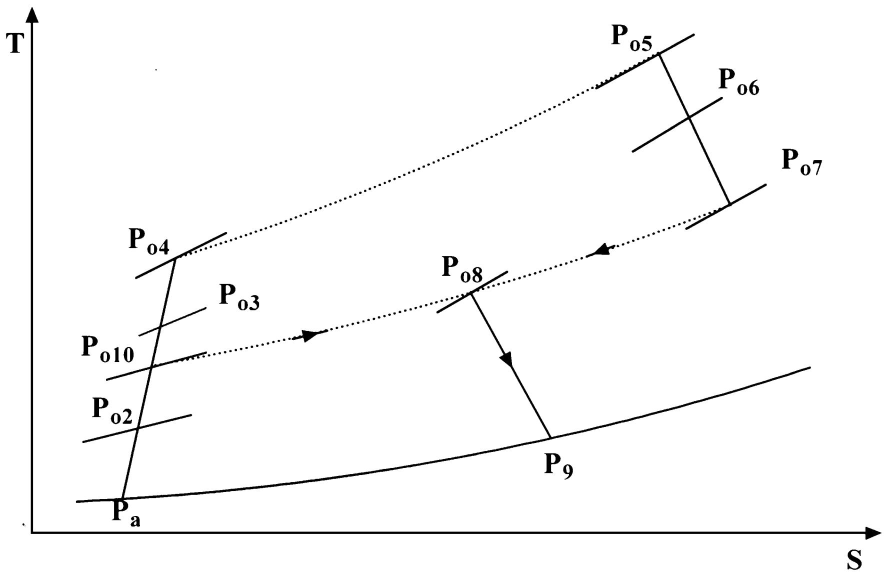


Fig. 6.16 Temperature–entropy diagram for two-spool mixed turbofan

In some engines like the CFM56 series, the mixing process takes place in a mixer preceding the nozzle. It results in a quieter engine than if the mixer was not present.

1. Energy balance for the low-pressure spool

Considering a mechanical efficiency for the low-pressure spool of (η_{m1}), then:

$$W_{\text{fan}} + W_{\text{LPC}} = \eta_{m1} \lambda_1 W_{\text{LPT}}$$

or

$$\dot{m}_a (1+\beta) Cp_c (T_{010} - T_{02}) + \dot{m}_a Cp_c (T_{03} - T_{010}) \\ = \eta_{m1} \lambda_1 \dot{m}_a (1+f) Cp_h (T_{06} - T_{07}) \tag{6.59}$$

2. Energy balance for the high-pressure spool

Also a second mechanical efficiency for the high-pressure spool is (η_{m2}) assumed:

$$W_{\text{HPC}} = \eta_{m2} \lambda_2 W_{\text{HPT}}$$

thus:

$$\dot{m}_a Cp_c (T_{04} - T_{03}) = \eta_{m2} \lambda_2 \dot{m}_a (1+f) Cp_h (T_{05} - T_{06}) \tag{6.60}$$

3. Mixing process

The hot gases leaving the low-pressure turbine and the cold air leaving the fan bypass duct are mixed, giving new properties at state (8). Thus, such a process is governed by the first law of thermodynamics as follows:

$$H'_{03} + H_{07} = H_{08}$$

$$\beta \dot{m}_h Cp_c T_{03} + \dot{m}_h (1+f) Cp_h T_{07} = \dot{m}_h (1+\beta+f) Cp_h T_{08}$$

which is reduced to:

$$\beta Cp_c T_{03} + (1+f) Cp_h T_{07} = (1+\beta+f) Cp_h T_{08} \tag{6.61}$$

Now for a better evaluation of the gas properties after mixing, we can use mass-weighted average properties of the gases at state (8) as follows:

$$Cp_8 = \frac{(1+f)Cp_7 + \beta Cp_3}{1+f+\beta}$$

$$R_8 = \frac{(1+f)R_7 + \beta R_3}{1+f+\beta}$$

$$\gamma_8 = \frac{Cp_8}{Cp_8 - R_8}$$

Consider the real case of mixing where normally losses are encountered and a pressure drop is associated with the mixing process. Such pressure losses are either given as the value ΔP_{mixing} or as a ratio r_m in the mixing process:

$$P_{08} = P_{07} - \Delta P_{\text{mixing}} \qquad \text{or}$$

$$P_{08} = r_{\text{m}} P_{07} \qquad \text{where } r_m < 1 \approx 0.98$$

Check nozzle choking as previously outlined. If the nozzle is unchoked, then the exhaust velocity is calculated from the relation:

$$V_9 = \sqrt{\frac{2\gamma_c R T_{08} \eta_n}{(\gamma_c - 1)}\left[1 - (P_a/P_{08})^{\frac{\gamma_c - 1}{\gamma_c}}\right]}$$

If the nozzle is choked, then $V_9 = \sqrt{\gamma_c R T_9} = \sqrt{\frac{2\gamma_c}{\gamma_c+1} R T_{08}}$

The thrust force is then given by the relation:

$$T = \dot{m}_a\left[(1 + f + \beta)V_9 - (1 + \beta)U\right] + A_9(P_9 - P_a) \tag{6.62}$$

6.3.5.2 Mixed Turbofan with Afterburner

Since more than three decades, most military aircrafts are powered by a low-bypass ratio turbofan fitted with an afterburner [15]. One of the earliest afterburning turbofan engines is the Pratt & Whitney TF30 that powered the F-111 and the F-14A Tomcat. First flight of the TF30 was in 1964 and production continued until 1986. Afterburning gives a significant thrust boost for takeoff particularly from short runways like air carriers, transonic acceleration, and combat maneuvers, but is very fuel intensive. This engine is a mixed low-bypass ratio forward fan one. The mixed flow still has a sufficient quantity of oxygen for another combustion process. Thus, an afterburner is installed downstream of the low-pressure turbine and upstream of the nozzle. Tremendous amounts of fuel are burnt in the afterburner when it is lit. This rises the temperature of exhaust gases by a significant amount which results in a higher exhaust velocity/engine specific thrust. For a turbofan engine, afterburning (or reheat) offers greater gains because of the relatively low temperature, after mixing of the hot and cold streams and the large quantity of the excess air available for combustion. Exhaust nozzle is normally of the variable area type to furnish a suitable media for different operating conditions. Unlike the main combustor, an afterburner can operate at the ideal maximum (stoichiometric) temperature (i.e., about 2100 K). Now, at a fixed total applied fuel-to-air ratio, the total fuel flow for a given fan airflow will be the same, regardless of the dry specific thrust of the engine. However, a high specific thrust turbofan will, by definition, have a higher nozzle pressure ratio, resulting in a higher afterburning net thrust and, therefore, lower afterburning specific fuel consumption.

Though afterburning turbofan engines are mostly two-spool engine, some few single-spool ones are found. An example for single-spool afterburning is the SNECMA M53 developed for the Dassault Mirage 2000 fighter. The engine is in service with different air forces, including the latest Mirage 2000–5 and 2000–9 multirole fighters. It had two variants, M 53–5 and M63-P2, which develop dry thrust, 54.0–64.7 kN, and wet (afterburning thrust) of 86.3–95.1 kN. Overall pressure ratio is 9.8:1, bypass ratio is 0.36:1, specific fuel consumption is 0.90 (kg/daN.h), and thrust-to-weight ratio is 6.5.

Examples for other two-spool afterburning turbofans are:

The Pratt & Whitney series F100-220 (thrust: 40 K), F100-229 (48 K), F100-232 (28 K), F119 (65 K)
Eurojet EJ200
General Electric F110
Rolls-Royce Adour Mk.104
Russian engine RD-133

It is interesting to add here that military engines are now so powerful that the latest fighters can *supercruise*; (flying at sustained supersonic speed without the use of the afterburner). Examples are the Lockheed Martin: F-22 Raptor and Eurofighter: Eurofighter Typhoon can accelerate to supersonic speed (without using augmentation) and then sustain such a speed indefinitely in dry thrust. F-22 can supercruise at Mach 1.82, while Eurofighter Typhoon can supercruise at Mach 1.1–1.5.

A. Forward fan mixed afterburning low-bypass turbofan (LBPT) engine

The flow mixing will be the only difference in the cycle from that of afterburning turbojet engine. Figure 6.17 shows the configuration of a single-spool afterburning turbofan engine, while Fig. 6.18 shows its T-s diagram, and Fig. 6.19 shows its block diagram together with its input and output data.

A typical layout of a two-spool afterburning turbofan engine is displayed in Fig. 6.20, while its T-s diagram for an ideal cycle is shown in Fig. 6.21.

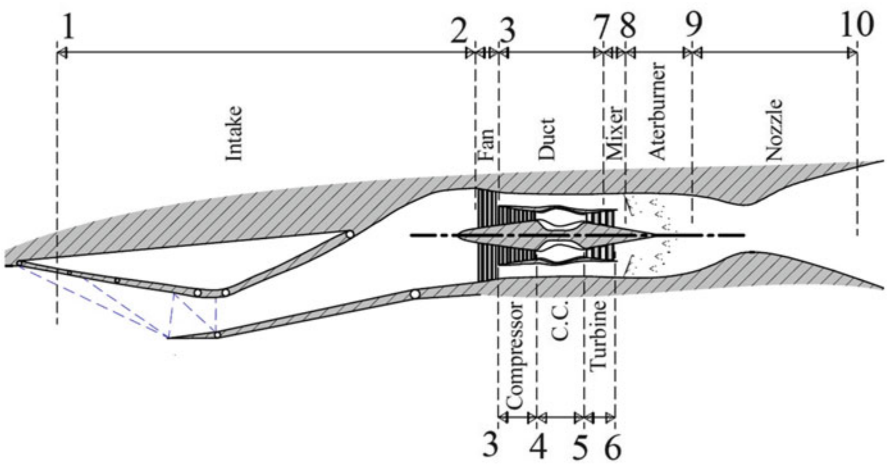


Fig. 6.17 Configuration of mixed turbofan with afterburner

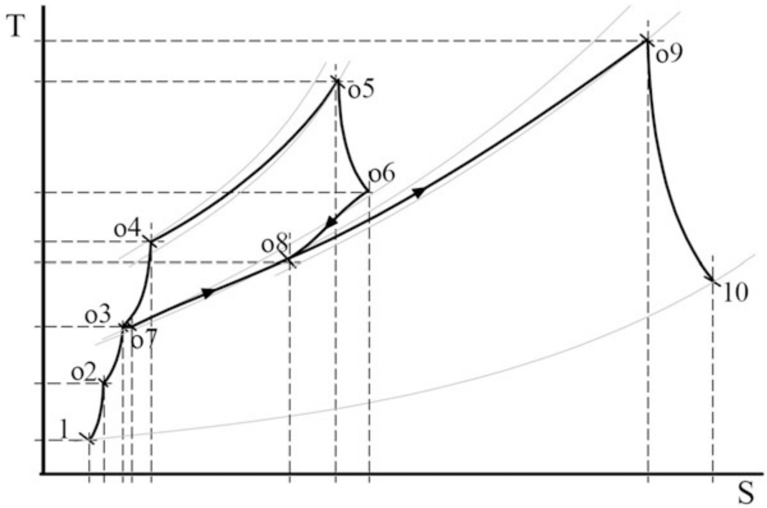


Fig. 6.18 T-s diagram for a single-spool mixed turbofan with afterburner

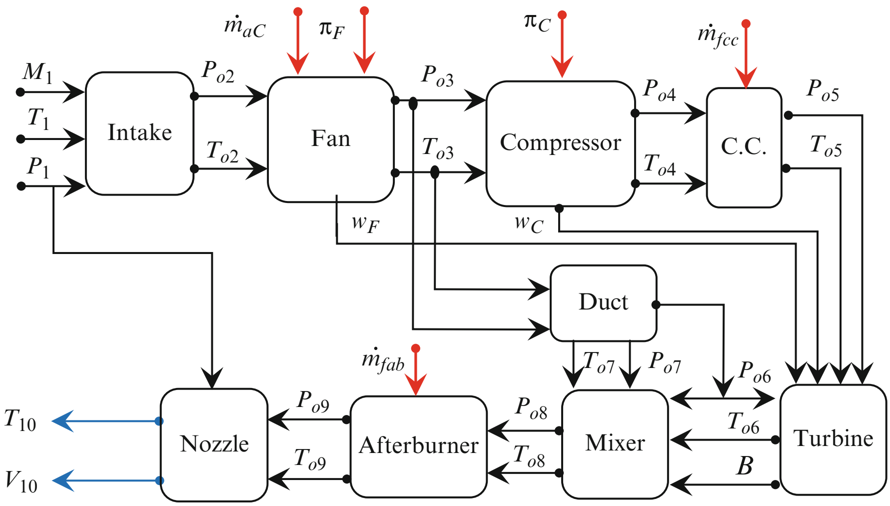


Fig. 6.19 Block diagram for a single-spool mixed afterburning turbofan engine

The same conditions necessary for mixing described above still hold. Thus, the following relation stands for ideal conditions (no pressure losses):

$$P_{010} = P_{07} = P_{08} = P_{011}$$

Normally mixing process is associated with pressure losses, which may be governed by either one of the following equations for losses:

$$P_{08} = P_{07} - \Delta P_{\text{mixing}}$$

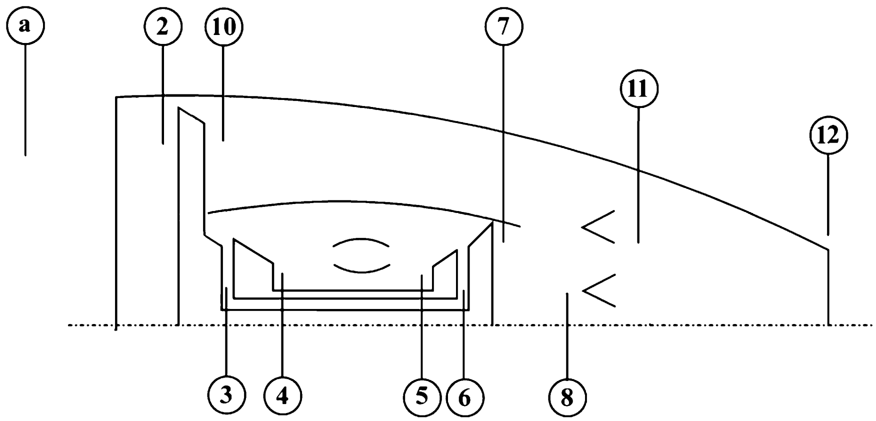


Fig. 6.20 Layout of a typical two-spool mixed afterburning engine

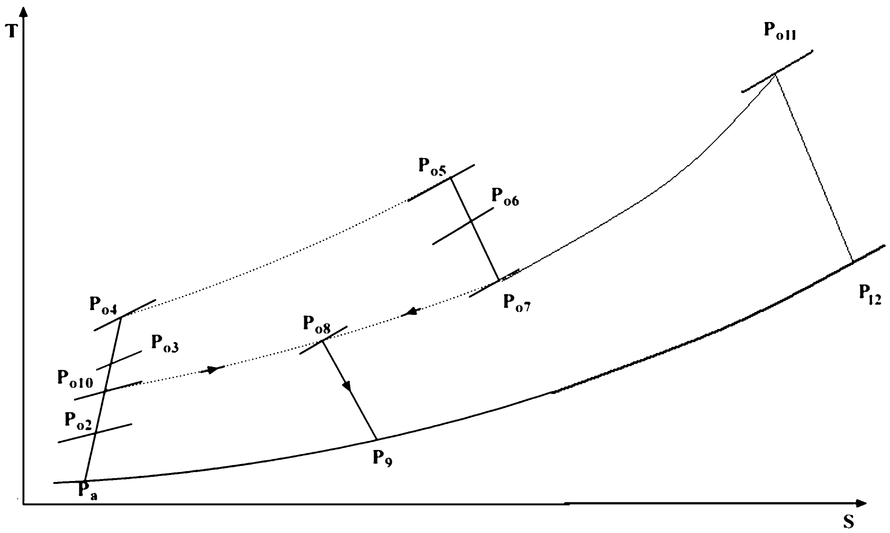


Fig. 6.21 T-s diagram for two-spool mixed afterburning engine

$$\text{Or} \qquad P_{08} = r_m P_{07} \qquad where\, r_m < 1 \approx 0.98$$

Here P_{011} is the total pressure at the outlet of the afterburner, when it is operative. Moreover, the outlet temperature of the afterburner is T_{011} which is known in advance. It is limited by the maximum temperature that the material of afterburner can withstand. However, it could be much higher than the total turbine inlet temperature (TIT) due to the absence of mechanical stresses arising from centrifugal forces created by the rotation of the turbine blades.

Now the mass balance in the afterburner gives the following afterburner fuel-to-air ratio (f_{ab}), where (f_{ab}) defined as:

$$f_{\text{ab}} = \frac{\dot{m}_{fab}}{\dot{m}_c + \dot{m}_h} \tag{6.63a}$$

Then the energy balance of the afterburner yields the relation:

$$f_{\text{ab}} = \frac{Cp_{11}T_{011}\text{-}Cp_8T_{08}}{\eta_{ab}Q_{HV} - Cp_{11}T_{011}} \tag{6.63b}$$

The pressure loss in the afterburner is also governed by the relation:

$$P_{09A} = P_{08} - \Delta P_{ab}$$

Next a convergent–divergent nozzle will enhance a full expansion to the ambient pressure. Thus, the jet speed will be given by the relation:

$$\therefore V_{12} = \sqrt{2Cp_{11}T_{011}\left[1 - \left(\frac{P_a}{P_{011}}\right)^{\frac{\gamma_{11}-1}{\gamma_{11}}}\right]} \tag{6.64}$$

The thrust force is now:

$$T = \dot{m}_e V_{12} - \dot{m}_a V_\infty$$

With the exhaust mass flow rate defined as:

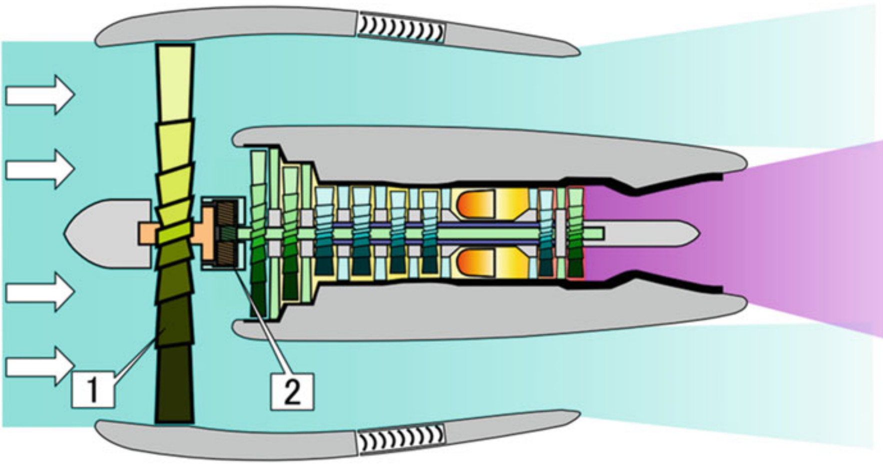
$$\dot{m}_e = m_c + \dot{m}_h(1+f) + \dot{m}_{f_{ab}}$$
$$\dot{m}_e = \{\dot{m}_c + \dot{m}_h(1+f)\}(1+f_{ab})$$

Then the thrust is now expressed as:

$$T = \{\dot{m}_c + \dot{m}_h(1+f)\}(1+f_{ab})V_{12} - (\dot{m}_c + \dot{m}_h)V_\infty \tag{6.65}$$

6.3.5.3 Geared Turbofan (GTF)

Geared turbofans are of the two-spool unmixed forward fan category (Figs. 6.22 and 6.23). Since the fan is normally part of the low spool, both fan and LPT are turning at the same speed (N_1). This speed, however, often is a compromise. The fan really operates more efficiently at low rotational speeds (rpm), while the rest of the low spool is more efficient at higher speeds. Putting a reduction gear in between these components makes it possible for the fan and the low spool to run at their



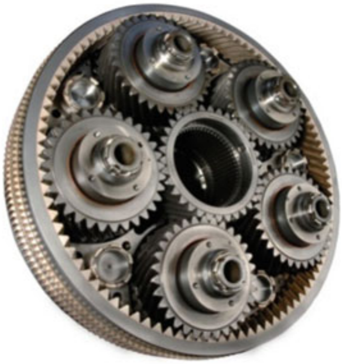


Fig. 6.22 Layout of geared turbofan (*top*) and planetary gearbox (*bottom*). (*1*) Fan (*2*) planetary gearbox

optimum speed, namely, N_1 for fan and N_3 for LPT. This in turn will minimize the possibility of formation of shock waves in the fan leading to a higher efficiency. The other advantages are more fuel efficiency, reduced noise levels, reduced emission (CO_2 and NO_x), fewer engine parts (fewer compressors and turbine stages), as well as reduction in overall operating costs. Gearbox used here is of the planetary type as shown in Fig. 6.22. Planetary gearbox is recommended as its length is short to match with minimum size requirements of turbofan engines.

Examples of geared turbofan engines

Low-pressure spool may be either composed of a fan driven by low-pressure turbine or a fan and low-pressure compressor representing the compression section driven by a low-pressure turbine. In either cases, a reduction gearbox is located directly downstream the fan.

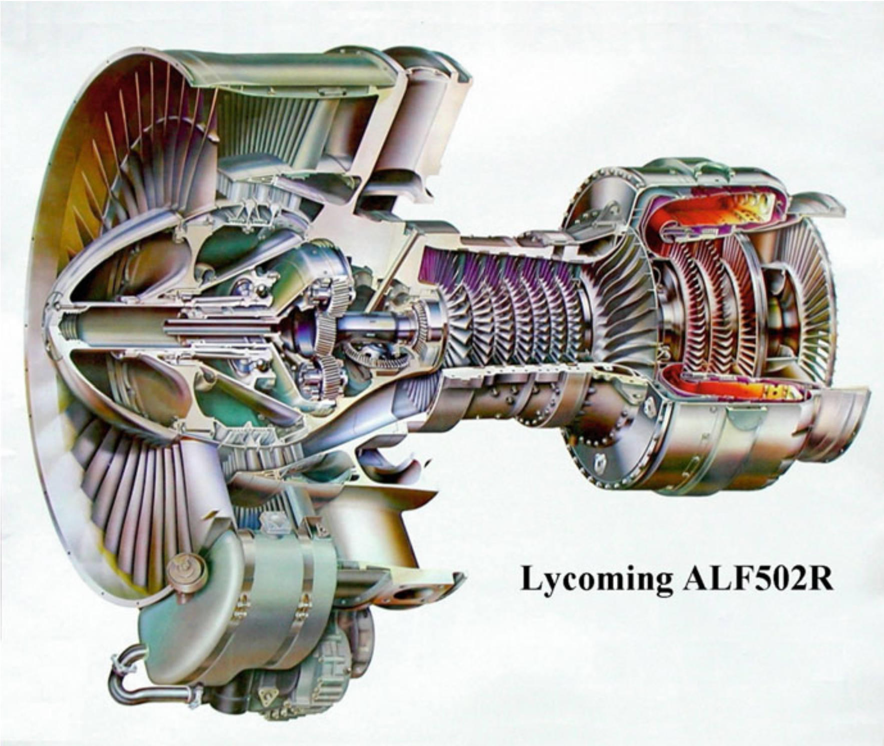


Fig. 6.23 Lycoming ALF502R geared turbofan

An example for geared turbofan is the *Honeywell LF 507* produced by Lycoming, AlliedSignal, and then Honeywell Aerospace. The earlier ALF 502 was certified in 1980 [16]. The improved, higher-thrust LF 507 was used on the Avro RJ update of the British Aerospace BAe 146. Lycoming ALF502R shown in Fig. 6.23.

A second example is Pratt &Whitney PurePower PW1000G series (PW1100/1200/1400/1500/1700 G/1900) engine which represents the future geared turbofan scheduled for 2015–2018 [17]. It will power A320neo, MRJ, MS-21, and CSeries.

The PurePower PW1000G engine family has a maximum thrust in the range 62–100 kN.

It also has the following advantages;

It improves fuel burnup to 16 % versus today's best engines. That alone could save airlines nearly $1,400,000 per aircraft per year

It cuts carbon emissions by more than 3500 t per aircraft per year. That's equal to the effect of planting more than 900,000 trees.

It will be ready to power aircraft well with biofuels since it has successfully been tested with alternative fuel.

Its TALON™ X combustor slashes polluting emissions. Thus, it will surpass the most stringent standards (CAEP/6) by 50 % for nitrous oxide (NOx).

It slashes aircraft noise footprints by up to 75 %—a big relief to communities. At up to 20 dB below today's most stringent standard, it is the quietest engine in its class, meaning lower noise fees, shorter flight tracks, extended curfew operation, and quieter cabins.

Now, concerning thermodynamic analysis of geared turbofan, if the mechanical efficiency of gearbox is η_{gb}, then two cases are examined. Firstly, the low-pressure spool is composed of fan and a low-pressure turbine. The fan states are 2 and 3 and the LPT states are 6 and 7. Then energy balance is expressed by the relation:

$$(1+\beta)\dot{m}_a Cp_c(T_{03}-T_{02}) = \eta_{gb}\lambda_1\eta_{m1}\dot{m}_a(1+f-b)\ Cp_h(T_{06}-T_{07}) \tag{6.66}$$

where $b=\frac{\dot{m}_b}{\dot{m}_a}$ is the air bleed ratio defining the ratio between the air bled and the HPC to the core airflow rate.

In the second case, the low-pressure turbine (states 6 and 7) drives both fan (states 2 and 10) and low-pressure compressor (states 10 and 3). Then the energy balance equation will be expressed as:

$$\begin{aligned}(1+\beta)\dot{m}_a Cp_c(T_{010}-T_{02}) + \dot{m}_a Cp_c(T_{03}-T_{010})\\ = \eta_{gb}\lambda_1\eta_{m1}\dot{m}_a(1+f-b)\ Cp_h(T_{06}-T_{07})\end{aligned} \tag{6.67}$$

Concerning the high-pressure spool, with an air bled from a station just downstream of HPC and prior of combustion chamber:

$$\dot{m}_a Cp_c(T_{04}-T_{03}) = \lambda_2\eta_{m2}\dot{m}_a(1+f-b)\ Cp_h(T_{05}-T_{06}) \tag{6.68a}$$

Next, if *air is bled* from at a station within the high-pressure compressor (HPC), identified by the pressure P_{03b}, then the energy balance of the high-pressure spool is given by:

$$\begin{aligned}\dot{m}_a Cp_c(T_{03b}-T_{03}) + \dot{m}_a Cp_c(1-b)(T_{04}-T_{03b})\\ = \lambda_2\eta_{m2}\dot{m}_a(1+f-b)\ Cp_h(T_{05}-T_{06})\end{aligned} \tag{6.68b}$$

6.3.6 Forward Fan Unmixed Three-Spool Engine

The three-spool engine is composed of a low-pressure, intermediate-pressure, and high-pressure spools running at different speeds (N_1, N_2, and N_3). The fan and the low-pressure turbine (LPT) compose the low-pressure spool. The intermediate spool is composed of an intermediate-pressure compressor (IPC) and intermediate-pressure turbine (IPT). The high-pressure spool is also composed of a high-pressure compressor (HPC) and high-pressure turbine (HPT). Rolls-Royce was the first aero engine manufacturer to design, develop, and produce the three-spool turbofan engine. The Rolls-Royce RB211 was the first three-spool engine to enter service (1972). Later on several manufacturers developed and manufactured this type of engines.

The main advantages of three-spool arrangement:

1. Shorter modules and shafts which result in a shorted engine
   (a) Single-stage fan with no booster stages
   (b) Fewer overall compressor stages and fewer variable stages
   (c) Shorter high-pressure compressor
2. Higher efficiencies as each spool is running at its "optimum speed"
3. Greater engine rigidity
4. Lighter weight

The main drawbacks of this three-spool category are that they are more complex to build and maintain [18].

Examples for three-spool engines

1. *Rolls-Royce RB211*

The first of the whole series was RB211-22 series [18] *which first saw service in 1972.Its thrust rating is 169 kN. Several subsequent series were developed with thrust rating of 222–270 kN, featured FADEC (full authority digital engine—or electronics—control), and wide-chord fan (which increases efficiency, reduces noise, and gives added protection against foreign object damage).*

2. *Rolls-Royce Trent*

It is a family of high-bypass developed from RB211 with thrust ratings of 236–423 kN. Its layout provides lighter weight and better performance. It features the wide-chord fan and single crystal high-pressure turbine blades with improved performance and durability. The core turbomachinery is brand new, giving better performance, noise, and pollution levels.

The layout of this engine and the T-s diagram is shown in Figs. 6.24 and 6.25. Air bleed is also extracted from the high-pressure compressor at station (4b), not shown.

1. Energy balance of the 1st spool (fan and LPT)

$$(1+\beta)\dot{m}_a Cp_c(T_{03}-T_{02}) = \eta_{m1}\lambda_1\dot{m}_a(1+f-b)Cp_h(T_{08}-T_{09}) \tag{6.69}$$

2. Energy balance for the intermediate spool (IPC and IPT)

$$\dot{m}_a Cp_c(T_{04}-T_{03}) = \eta_{m2}\lambda_2\dot{m}_a(1+f-b)Cp_h(T_{07}-T_{08}) \tag{6.70}$$

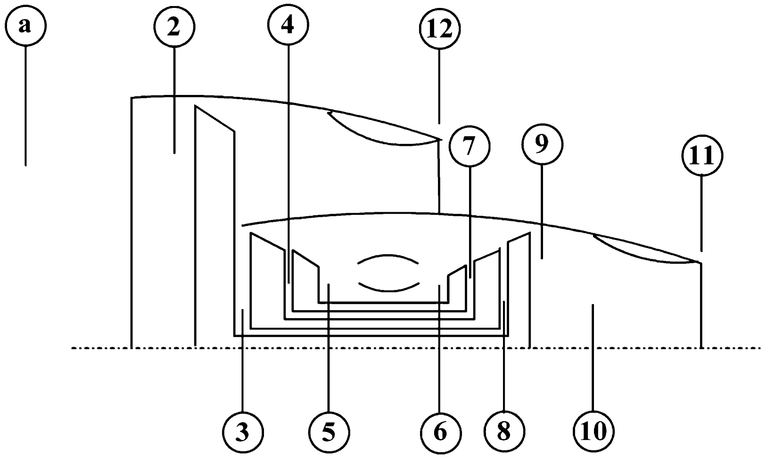


Fig. 6.24 Layout of a three-spool engine

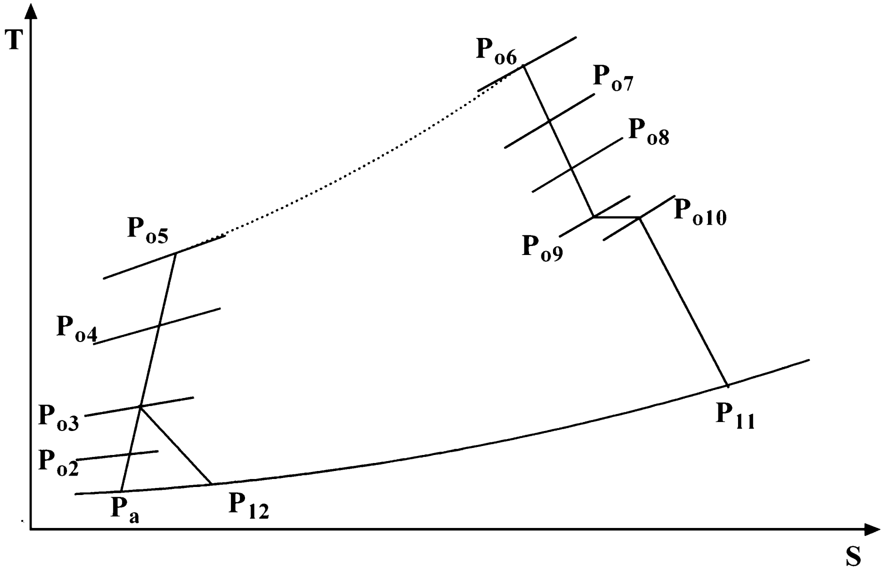


Fig. 6.25 T-s diagram for a three-spool turbofan

3. Energy balance for the high-pressure spool (HPC and HPT) with inter-bleed stage at (4b)

$$\begin{aligned} &\dot{m}_a Cp_c (T_{04b} - T_{04}) + (1-b)\dot{m}_a Cp_c (T_{05} - T_{04b}) \\ &\quad = \eta_{m3}\lambda_3 \dot{m}_a (1 + f - b) Cp_h (T_{06} - T_{07}) \end{aligned} \tag{6.71}$$

The same procedure discussed earlier in evaluating the fuel-to-air ratio, jet velocities of the cold air, and hot gases from the fan and turbine nozzles may be followed here. The propulsive, thermal, and overall efficiencies are obtained following the appropriate equations from Chap. 2.

Example 6.11 A. Triple-spool unmixed turbofan engine (Trent 700) is shown in Fig. 6.26. Bleed air having a percentage of 8 % is taken from HP compressor to cool HP and IP turbines as shown in Fig. 6.27. Engine has the following data in Table 6.4 below:

For takeoff operation where Mach number M = 0.2 at sea level, calculate:

1. The total temperature at the outlet of the fan and intermediate- and high-pressure compressors
2. The total temperature at the outlet of the high-, intermediate-, and low-pressure turbines

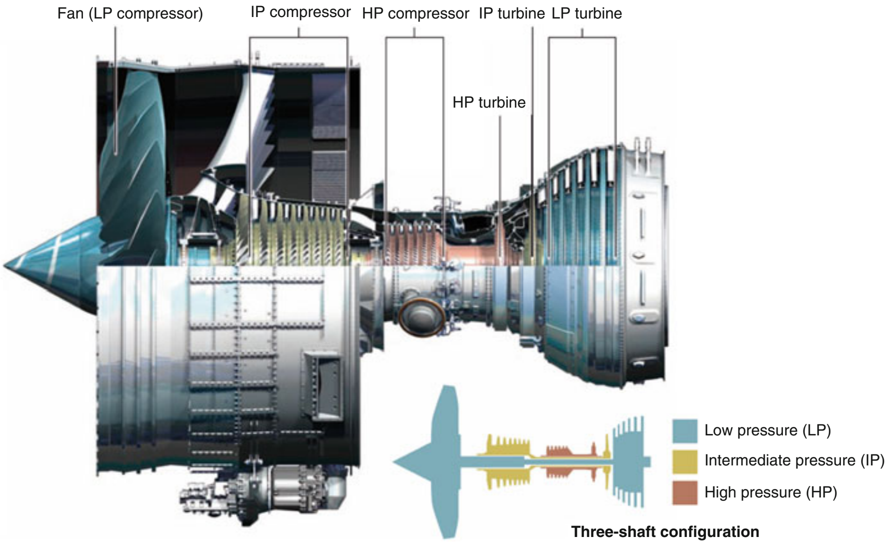


Fig. 6.26 Layout of unmixed three-spool engine (Trent 700)

Fig. 6.27 Layout of three-spool engine and air bleed details

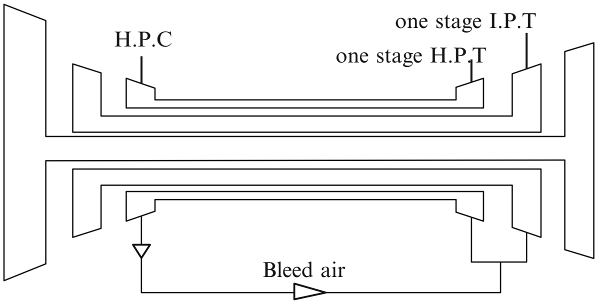


Table 6.4 A triple-spool unmixed turbofan engine operating conditions (Trent 700)

Variable	Value	Variable	Value
η_d	0.88	$\Delta p_{c.c.}$	0.03
η_f	0.9	$\Delta p_{fanduct}$	0
η_c	0.89	cp_c (J/kg.K)	1005
η_b	0.98	cp_h (J/kg.K)	1148
η_t	0.93	R (J/kg.K)	287
η_n	0.98	γ_c	1.4
η_m	0.99	γ_h	1.33
λ_1	0.84	B	0.08
λ_2	1	β	5.05
λ_3	1	TIT (K)	1543
M	0.82	π_{IPC}	5.8
π_F	1.45	π_{HPC}	4.2

3. The pressure and temperature at the outlet of cold and hot nozzles
4. Specific thrust, thrust specific fuel consumption, and propulsive, thermal, and overall efficiencies

Solution

At takeoff (M = 0.2) at sea level

Cycle analysis from Figs. 6.26 and 6.27:

Diffuser (a–2)

$$P_{02} = P_a\left(1 + \eta_d\frac{\gamma_c - 1}{2}M_a^2\right)^{\gamma_c/\gamma_c - 1} = 1.0384 \times 10^5 \ \text{Pa}$$

$$T_{02} = T_a\left(1 + \frac{\gamma_c - 1}{2}M_a^2\right) = 290.4653 \ \text{K}$$

Fan (2–3)

$$P_{03} = P_{02}\,\pi_f = \ 1.5057 \times 10^5\,\text{Pa}$$

$$T_{03} = T_{02}\left(1 + \frac{\pi_f^{\frac{\gamma_{c-1}}{\gamma_c}} - 1}{\eta_f}\right) = 326.6124\,\text{K}$$

Intermediate-pressure compressor (IPC) (3–4)

$$P_{04} = P_{03} \times \pi_{IPC} = 8.7333 \times 10^5 \ \text{Pa}$$

$$T_{04} = T_{03}\left(1 + \frac{\pi_{IPC}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{IPC}}\right) = 566.0401 \text{ K}$$

High-pressure compressor (HPC) (4–5)

Bleed air is extracted at an intermediate state defined as:

$$P_{04b} = P_{04} \times \sqrt{\pi_{\text{HPC}}} = 1.7898 \times 10^6\,\text{Pa}$$

$$T_{04b} = T_{04}\left(1 + \frac{\pi_{\text{HPC}}^{\frac{\gamma_c - 1}{2\gamma_c}} - 1}{\eta_{\text{HPC}}}\right) = 710.7556 \text{ K}$$

$$P_{05} = P_{04b} \times \sqrt{\pi_{\text{HPC}}} = 3.6680 \times 10^6$$

$$T_{05} = T_{04b}\left(1 + \frac{\pi_{\text{HPC}}^{\frac{\gamma_c - 1}{2\gamma_c}} - 1}{\eta_{\text{HPC}}}\right) = 892.4693\,\text{K}$$

Combustion chamber (CC) (5–6)

$$T_{06} = TIT, \quad P_{06} = P_{05} \times (1 - \Delta p_{c.c}) = 3.5579 \times 10^6 \text{ kPa}$$

$$f = (1 - b)\left(\frac{C_{ph} \times T_{06} - C_{pc} \times T_{05}}{\eta_b \times Q_R - C_{ph} \times T_{06}}\right) = 0.0213$$

High-pressure turbine (HPT) (6–7)

$$T_{07} = T_{06} - \left[\left(\frac{C_{pc}}{\eta_{m1} \times \lambda_1 \times (1 + f - b) \times C_{ph}}\right) \times ([T_{04b} - T_{04}] + (1 - b) \times [T_{05} - T_{04b}])\right]$$
$$= 1206.8 \text{ K}$$

$$P_{07} = P_{06}\left(1 - \frac{(T_{06} - T_{07})}{\eta_{\text{HPT}} \times T_{06}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} = 1.2134 \times 10^6 \text{ Pa}$$

Intermediate-pressure turbine (IPT) (7–8)

$$T_{08} = T_{07} - \left(\frac{C_{pc} \times [T_{04} - T_{03}]}{\lambda_2 \times \eta_{m2} \times \left(1 + f - \frac{b}{2}\right) \times C_{ph}}\right) = 995.3980 \text{ K}$$

$$P_{08} = P_{07}\left(1 - \frac{[T_{07} - T_{08}]}{\eta_{IPT} \times T_{07}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} = 5.2318 \times 10^5 \text{ Pa}$$

Low-pressure turbine (LPT) (8–9)

$$T_{09} = T_{08} - \left(\frac{(1+\beta) \times C_{pc} \times [T_{03} - T_{02}]}{\lambda_3 \times \eta_{m3} \times (1+f) \times C_{ph}}\right) = 798.3326 \text{ K}$$

$$P_{09} = P_{08}\left(1 - \frac{[T_{08} - T_{09}]}{\eta_{\text{LPT}} \times T_{o8}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} = 1.9938 \times 10^5 \text{ Pa}$$

Hot nozzle (10–11)

$$P_{010} = P_{09}(1 - \Delta p_{\text{duct}}) = 1.9339 \times 10^5 \text{ Pa}$$

To get critical pressure: $P_c = P_{010}\left(1 - \frac{1}{\eta_n}\left[\frac{\gamma_h - 1}{\gamma_h + 1}\right]\right)^{\frac{\gamma_h}{\gamma_h - 1}} = 1.0309 \times 10^5 \text{ Pa}$

Where $P_a = 1.01325 \times 10^5$ Pa

Since $P_C > P_a$, then the hot nozzle is choked:

$$\therefore P_{11} = P_c, \quad T_{11} = T_{010}\left(\frac{2}{\gamma_h + 1}\right) = 685.2640 \text{ K},$$

$$V_{11} = \sqrt{\gamma_h R T_{11}} = 511.4412 \text{ m/s}$$

Cold nozzle (3–12)

The critical pressure: $P_c = P_{03}\left(1 - \frac{1}{\eta_n}\left[\frac{\gamma_h - 1}{\gamma_h + 1}\right]\right)^{\frac{\gamma_h}{\gamma_h - 1}} = 7.8415 \times 10^4 \text{ Pa}$

Since $P_a = 1.01325 \times 10^5$ Paand then $P_C < P_a$, *the cold nozzle is unchoked*:

$$P_{12} = P_a, \qquad T_{12} = T_{03}\left[1 - \eta_n\left(1 - \left(\frac{P_a}{P_{03}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right)\right] = 292.3616 \text{ K},$$

$$V_{12} = \sqrt{2C_{ph}(T_{03} - T_{12})} = 280.4278 \text{ m/s}$$

Specific thrust

$$\frac{T}{m_a^\circ} = \left[\left(\frac{1}{1+\beta}\right)\left[(1+f)\times V_{11}+\beta\times V_{12}+(1+f)\left(\frac{RT_{11}}{P_{11}V_{11}}\right)(P_{11}-P_a)\right.\right.$$
$$\left.\left.+\beta\left(\frac{RT_{12}}{P_{12}v_{12}}\right)(P_{12}-P_a)\right]-V\right]$$
$$=253.4724\,\text{N.s/kg}$$

Where:

T is the thrust force, m_a° is the air mass flow rate.
V is the flight speed, V_{11} is the exhaust hot gas speed.
V_{12} is the exhaust cooled gas speed.

Since the cold nozzle is unchoked, then $P_{12}=P_a$, and the corresponding pressure thrust is zero.

Thrust specific fuel consumption

$$TSFC = \frac{f/(1+\beta)}{T/\dot{m}_a} = 1.3898\times10^{-5}\ \text{kg/(N.s)}$$

Propulsive efficiency

$$\eta_p = \frac{T/m_a^\circ\times V}{T/m_a^\circ\times V + \left[0.5\left(\frac{1+f}{1+\beta}\right)(V_{11}-V)^2\right] + \left[0.5\left(\frac{\beta}{1+\beta}\right)(V_{12}-V)^2\right]}$$
$$= 0.3275$$

Thermal efficiency

$$\eta_{th} = \frac{T/m_a^\circ\times V + \left[0.5\left(\frac{1+f}{1+\beta}\right)(V_{11}-V)^2\right] + \left[0.5\left(\frac{\beta}{1+\beta}\right)(V_{12}-V)^2\right]}{\left(\frac{f}{1+\beta}\right)Q_R} = 0.3560$$

Overall efficiency

$$\eta_o = \eta_p\times\eta_{th} = \frac{T/m_a^\circ\times V}{\left(\frac{f}{1+\beta}\right)Q_R} = 0.1166$$

6.4 Turbine-Based Combined-Cycle (TBCC) Engines

6.4.1 Introduction

Long distance airliner flights are troublesome to passengers particularly if it last for more than 6 h. This motivated aerospace industry for working hard in designing and manufacturing airplanes and space planes that will satisfy the greatest human demands for faster and comfortable Earth and space flights. This can be achieved by designing engines that power airplanes/space planes which fly at high speeds (Mach 3.0^{+}) and high altitudes ($20{,}000^{+}$ m). Such propulsion system has to facilitate low subsonic conditions for takeoff/climb flight segments and hypersonic speeds for cruise operation. Such tough requirements can be achieved only through combined-cycle engines (CCE) or sometimes identified as *hybrid engines*. These engines will combine a turbine-based cycle (TBC) (turbojet or turbofan) for low-to-moderate flight speed operation and a ram-based cycle (RBC) (ramjet, scramjet, or dual combustion scram) for high-speed conditions. These are also frequently identified as turbine-based combine cycle (TBCC) engines.

Moreover, such hybrid engines can also be employed in the long-range tactical missiles as well as high-performance unmanned aerial vehicles (UAVs) or unmanned combat aerial vehicles (UCAVs) [39].

Figure 6.28 illustrates such a hybrid engine powering a future super-/hypersonic speed transport.

Figure 6.29a illustrates hybrid engine layout for low-/moderate-speed operation, where turbojet (or turbofan) engine is active, while ramjet/scramjet engine is inoperative. Figure 6.29b illustrates operative ramjet/scramjet engine and inoperative turbojet/turbofan engine during high-speed operation.

Concerning turbine-based cycle, only afterburning category of turbojet or turbofan engines is employed. However, all types of ram, scram, or dual combustion scram are employed as ram-based cycles. These two groups will generate the following six different combinations for the TBCC:

Turbojet and ramjet (TJRJ)
Turbojet and scramjet (TJSJ)
Turbojet and dual combustion scramjet (TJDJ)
Turbofan and ramjet (TFRJ)
Turbofan and scramjet (TFSJ)
Turbofan and dual combustion scramjet (TFDJ)

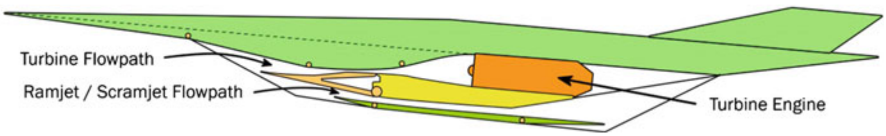


Fig. 6.28 Layout of a hybrid or turbine-based combined-cycle engine (TBCC)

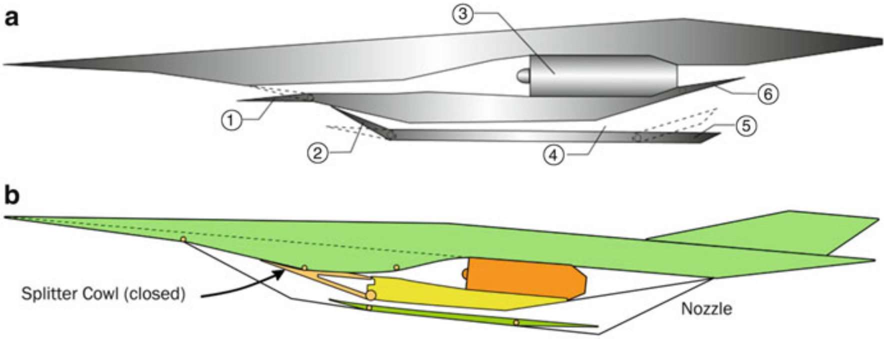


Fig. 6.29 Layout of a turbine-based combined-cycle engine (TBCC). (**a**) Operative turbojet/turbofan only. (**b**) Operative ramjet (scramjet) only

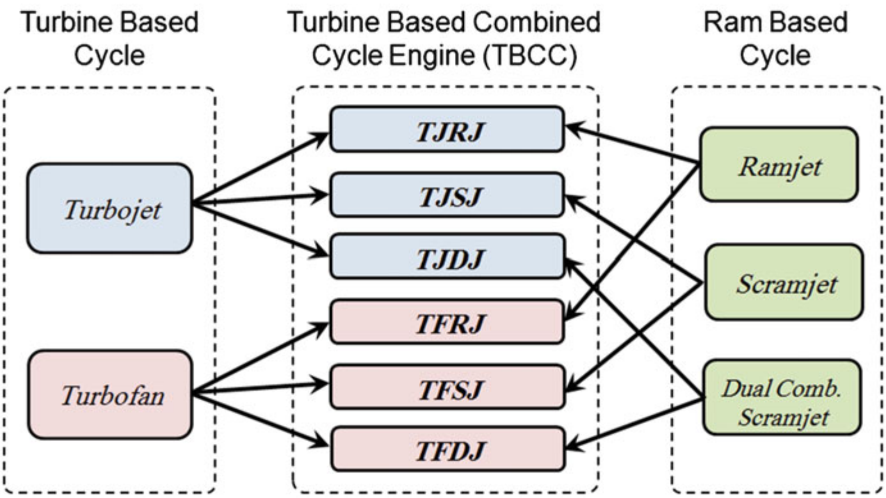


Fig. 6.30 Block diagram of the different configurations of the turbine-based combined cycle [19]

Figure 6.30 shows block diagram of these different combinations of the turbine-based combined cycle [19].

It is worth mentioning here that for an aircraft cruising at Mach 6, a *multistage vehicle* is an alternative for TBCC (or hybrid) engine. A multistage vehicle is composed of two vehicles (or stages), one having turbine engines to enhance takeoff and acceleration to supersonic speeds where the second vehicle (stage) is driven by ramjet or scramjet engines for supersonic/hypersonic flight speeds (Fig. 6.31).

In the following sections, a brief historical review of super-/hypersonic aircrafts followed by mathematical modeling of such complex engine will be given.

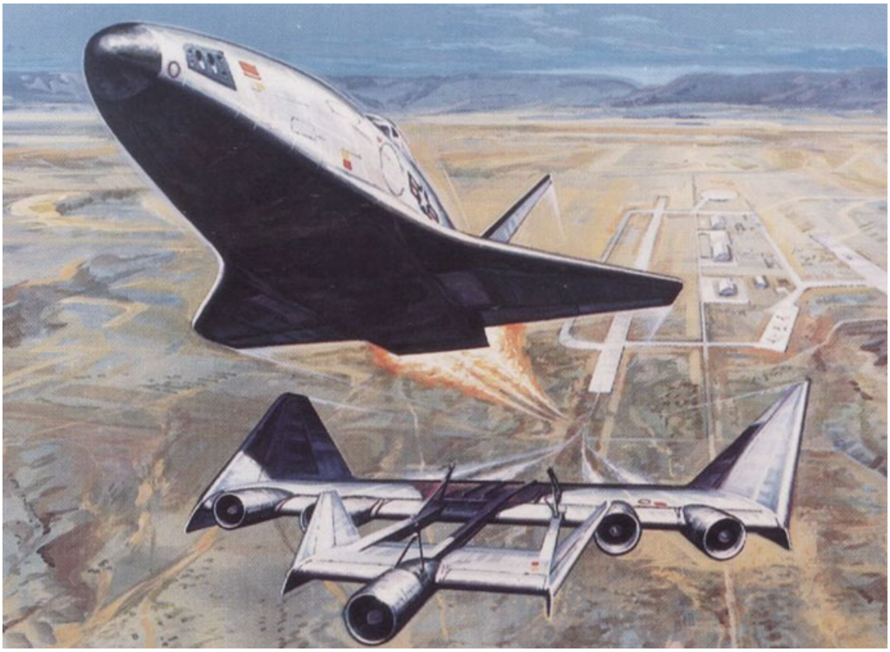


Fig. 6.31 A multistage aircraft concept

6.4.2 Historical Review of Supersonic and Hypersonic Aircrafts

6.4.2.1 Supersonic Aircrafts and Programs

Supersonic aircrafts have been used almost entirely for research and military purposes. Only two airliners, Concorde and the Tupolev Tu-144, were ever designed for civil use.

Quick highlights will be described as:

On October 14, 1947, the Bell X-1 rocket plane, launched at an altitude of approximately 20,000 ft from the Boeing B-29, became the first airplane to fly faster than the speed of sound [20]. The X-1 reached a speed of 1127 km/h (Mach 1.06) at an altitude of 43,000 ft. However, it was powered by a liquid-fueled *rocket*.

In 1954, the second generation of Bell X-1 reached a Mach number of 2.44 at an altitude of 90,000 ft.

On December 22, 1964, Lockheed SR-71 Blackbird (Fig. 6.32) entered service. It is a long-range, strategic reconnaissance aircraft which can fly at Mach number of (3+), maximum speed (3530+) km/h at 80,000 ft (24,000 m). It is powered by 2 Pratt & Whitney J58-1, *turboramjet* engines continuous-bleed afterburning turbojet [21]. However, it retired on October 9, 1999.

The Tupolev Tu-144 was the first commercial supersonic transport aircraft (SST), Fig. 6.33. The design, publicly unveiled in January 1962, was constructed

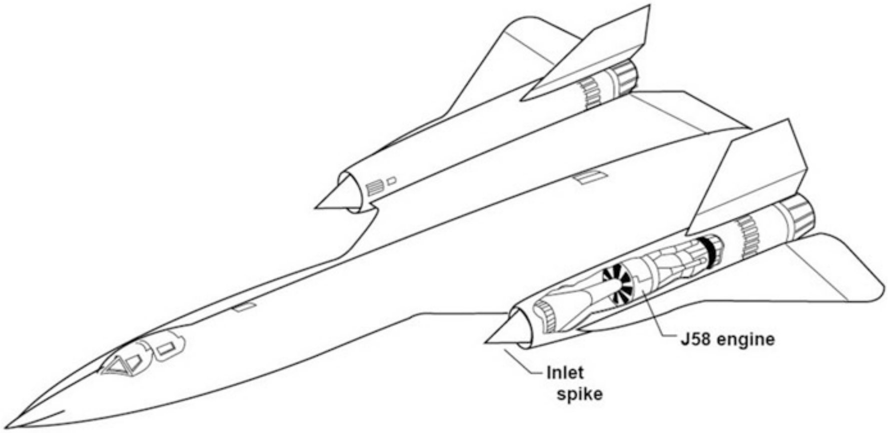


Fig. 6.32 SR-71 aircraft [21]

Fig. 6.33 Tupolev TU-144

in the Soviet Union under the direction of the Tupolev design bureau, headed by Alexei Tupolev. The prototype first flew on *December 31, 1968,* near Moscow, two months before the first flight of Concorde. The Tu-144 first went supersonic on June 5, 1969, and, on May 26, 1970, became the first commercial transport to exceed

Fig. 6.34 Concorde supersonic airplane

Mach 2. Tu-144 is powered by 4 × Kolesov RD-36-51 turbojet, 200 kN (44,122 lbf) each. It was withdrawn from service 1985 after only a ten-year line life. However, in 1994, NASA had some developments for TU-144 that led to the first flight of the flying laboratory plane TU-144LL on November 29, 1996. The last time TU-144 was shown was during MAKS 2013 Eleventh International Aviation and Space Salon held from August 27 to September 1, 2013, in Zhukovsky, Moscow region.

Aérospatiale-BAC Concorde was jointly developed and produced by Aérospatiale and the British Aircraft Corporation (BAC) (Fig. 6.34). On March 2, 1969, "Concorde" French prototype 001 made its first flight from Toulouse, while on April 9, 1969, "Concorde" British prototype 002 made its first flight from Filton. Concorde entered service in *1976* and continued commercial flights until November 26, 2003. Concorde powerplant is 4 × Rolls-Royce/SNECMA Olympus 593 Mk 610 afterburning turbojets having dry thrust 140 kN each while thrust with afterburner 169 kN each.

Many projects for *SST* were cancelled. Examples are:

(British) Bristol Type 223; (French–American) Convair BJ-58-9, Boeing 2707–100, Lockheed L-2000, Douglas 2229, and Tupolev Tu-244; (Russian–American) Sukhoi-Gulfstream S-21

Supersonic business jets (SSBJ) are a proposed class of small supersonic aircraft. None has yet flown. Examples are Aerion SBJ, HyperMach SonicStar, Tupolev Tu-444, and Gulfstream X-54. All are still under development.

Examples for *flying strategic bombers* (defined as a heavy bomber that carries a large bomb load over long distances) are Convair B-58A Hustler (1956), Tupolev

Tu-22 (1959), Myasishchev M-50 (1959), North American XB-70 Valkyrie (1964), Tupolev Tu-22M (1969), Sukhoi T-4 (1972), and Rockwell B-1B Lancer (1986). A next-generation stealthy supersonic strategic bomber is being planned in the USA under the 2037 Bomber project.

Few examples for *flying supersonic fighters* (sometimes called fast jets) are listed hereafter and arranged by country:

China
Shenyang J-11 (1998), Shenyang J-15 Flying Shark (2009), Shenyang J-16 (2012)
Egypt
Helwan HA-300 (March 7, 1964) and retired May 1969
France
Dassault Mirage 5 (1967), Dassault Mirage 2000 (1978), Dassault Rafale (1986)
Japan
Mitsubishi T-2 (1971), Mitsubishi F-1 (1975), Mitsubishi F-2 (1995)
Sweden
Saab 32 Lansen (1952), Saab 35 Draken (1955), Saab JAS 39 Gripen (1988)
Soviet Union/Russia
Mikoyan MiG-31 (1975), Mikoyan MiG-35 (2007), Sukhoi Su-35 (2008)
United States
North American F-100 Super Sabre (1953), Lockheed Martin F-22 Raptor (1997), Lockheed Martin F-35 Lightning II (2006)

It is to be noted that all the above listed aircrafts are powered by either one or more of turbojet or turbofan engines fitted with afterburners.

6.4.2.2 Hypersonic Vehicles

Aircraft or spacecrafts flying at speeds above Mach 5 are often referred to as hypersonic aircraft. A very brief review of the work held in the field of hypersonic aviation in different countries is given in [22]. NASA has been working in the design of a lot of hypersonic commercial aircrafts from the 1970s till now. Perhaps one of the most well-known activity concepts is the commercial derivative of the NASP project.

Concerning military applications, trends of the 1950s and 1960s indicated that military aircraft had to fly faster and higher to survive, so concepts for high-altitude fighters and bombers cruising at Mach 4 or more were not uncommon. Although the trend soon fizzled and military planners looked to maneuverability and stealth for survival, the military has recently shown renewed interest in hypersonic flight. For example, many have conjectured about the existence of a Mach 5 spy plane, the Aurora, that may be a scramjet powered (Fig. 6.35).

The only manned aircraft to fly in the low hypersonic regime was the X-15 (Fig. 6.36) and the Space Shuttle during reentry. American Robert White flew the X-15 rocket-powered research aircraft at Mach 5.3 thereby becoming the first pilot

Fig. 6.35 Aurora a hypersonic fighter

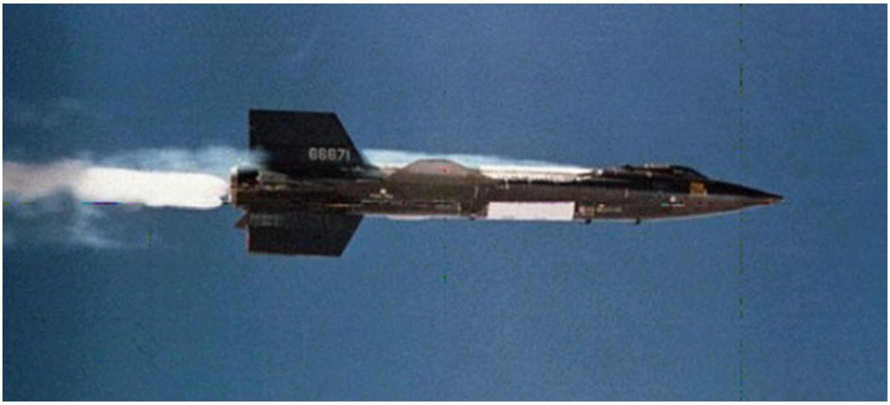


Fig. 6.36 X-15

to reach hypersonic velocities and at a record altitude of 314,750 ft on July 17, 1962. White's mark was later topped by fellow X-15 pilot Pete Knight who flew the craft to a maximum speed of Mach 6.72 in 1967. The record still stands today as the highest velocity ever reached by an aircraft. Recently, an unmanned X-43A used a scramjet, to make two hypersonic flights, one at Mach 7, the other at

Mach 10. Because lift and drag depend on the square of the velocity, hypersonic aircraft do not require a large wing area.

6.4.3 Technology Challenges of the Future Flight

The key technology challenges that are derived from the customer requirements and vehicle characteristics are related to economics, environment, or certification [23]:

Environment
Benign effect on climate and atmospheric ozone
Low landing and takeoff noise
Low sonic boom
Economics range, payload, fuel burn, etc.
Low weight and low empty weight fraction
Improved aerodynamic performance
Highly integrated airframe/propulsion systems
Low thrust specific fuel consumption (*TSFC*)
Long life
Certification for commercial operations
Acceptable handling and ride qualities
Passenger and crew safety at high altitudes
Reliability of advanced technologies, including synthetic vision
Technical justification for revising regulations to allow supersonic operations over land

6.4.4 Propulsion System Configurations

6.4.4.1 Introduction

There are two main layouts for the turboramjet (or TBCC) engine, namely:

Wraparound configuration
Over–under configuration

The differences between them are:

The position of the ram with respect to turbojet
The position of the afterburner of the turbojet with respect to the ram
Wraparound turboramjet

The wraparound installation methods are limited for axisymmetric propulsion systems. In such a configuration, the turbojet is mounted inside a ramjet (Fig. 6.37).

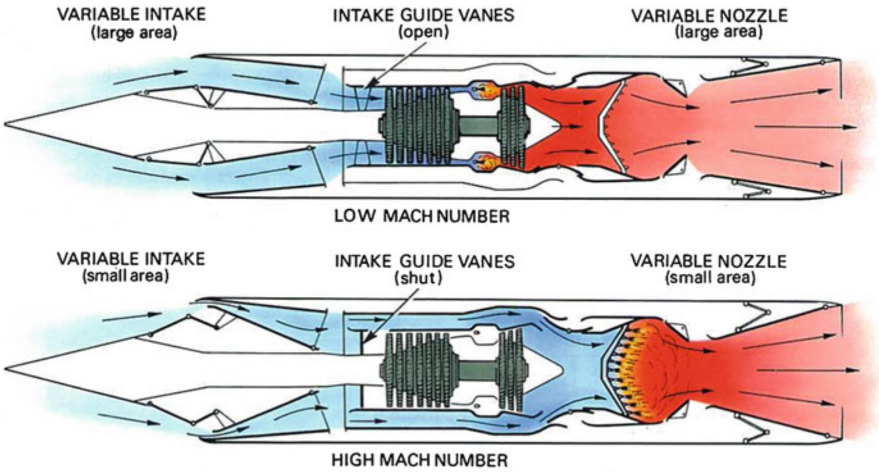


Fig. 6.37 Wraparound turboramjet [26]

Usually both engines share the same intake and nozzle, while the combustion chamber of the ramjet engine may be located in the afterburner of the turbojet engine or separated from it. The flow is divided between the two engines by mean of a number of bypass flaps or movable inlet guide vanes—located just downstream of the diffuser—that control the air mass fraction passing through each engine. The wraparound configuration with ram burner in the afterburner of turbojet was used in the Pratt & Whitney J58 engine that powered the Blackbird SR-71 that flows with speed of Mach 3+ (Fig. 6.32). Figure 6.37 shows wraparound TBCC installed on SR-71 in its two configurations for low and high Mach number operation [26]. During low-speed flight, these controllable flaps close the bypass duct and force air directly into the compressor section of the turbojet. During high-speed flight, the flaps block the flow into the turbojet, and the engine operates like a ramjet using the aft combustion chamber to produce thrust.

The Nord 1500 Griffon II, a French aircraft powered by a SNECMA ATAR 101 E-3 dry turbojet and a wraparound ramjet having a separate combustion chamber, was the first aircraft powered by combined-cycle engine in the 1950s (Fig. 6.38).

Two installation configurations are seen for the wraparound turbo ramjet engines, namely:

Wing like SR-71 [25–27] (Fig. 6.37)
Fuselage like Griffon II (Fig. 6.38)

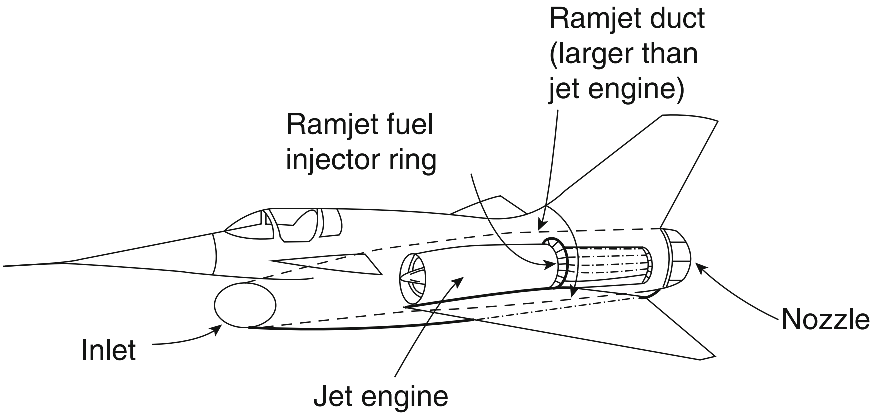


Fig. 6.38 Cutaway in the Nord 1500 Griffon II powered by wraparound TBCC

6.4.4.2 Over–Under Turboramjet

In over–under configuration, the engines are installed one over the other, while the vehicle body serves as part of the intake and the nozzle. In this configuration, the combustion chambers are totally separated. However, two cases are encountered for intakes and nozzles, namely:

(A) Separate intakes and nozzles (Fig. 6.39)
(B) Shared intake and nozzle (Fig. 6.40)

The ram-based cycle has a rectangle cross section. The intake(s) and nozzle (s) are rectangle, while they may be shared between the engines. In shared intake and nozzle, the system should contain flaps in the intake to divide the incoming airflow between the two engines and other flaps in nozzle to control the exit area of each engine.

In the late 1980s, the NASA Lewis Research Center discussed analytically and experimentally the performance of over–under TBCC configuration with separated intakes and nozzles and the interaction between the two intakes. Data from this study was presented in [28].

The NASA Lewis Research Center in the mid-1990s extended its studies for over–under TBCC configuration with shared intake and nozzle. Non-afterburning turbojet engines are integrated with a single-throat ramjet. The result is a simple, lightweight system that is operable from takeoff to Mach number 6. Flow in intake, spillage drag, system performance, and modes of operation were discussed in a series of publications ([29] and [30]).

Concerning installation of over–under layouts, the following types are seen:

Fuselage installation (Fig. 6.41)
Wing installation (Fig. 6.42)
Tail installation (Fig. 6.43)

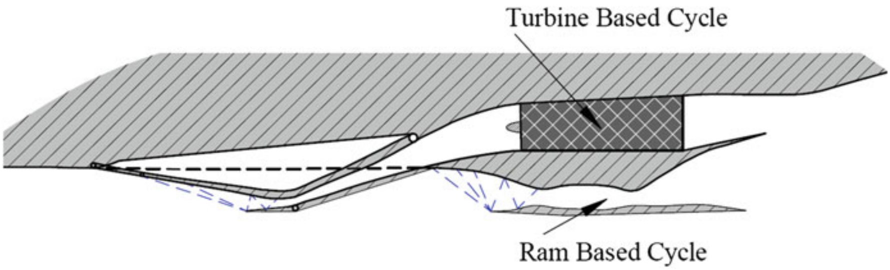


Fig. 6.39 Over–under TBCC configuration with separate intake and nozzle

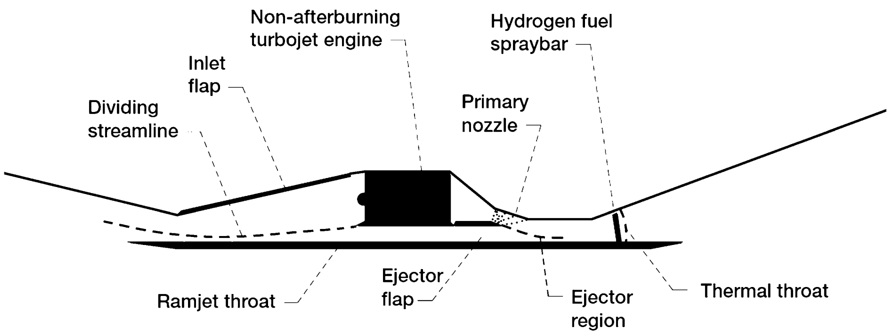


Fig. 6.40 Over–under TBCC configuration with shared intake and nozzle

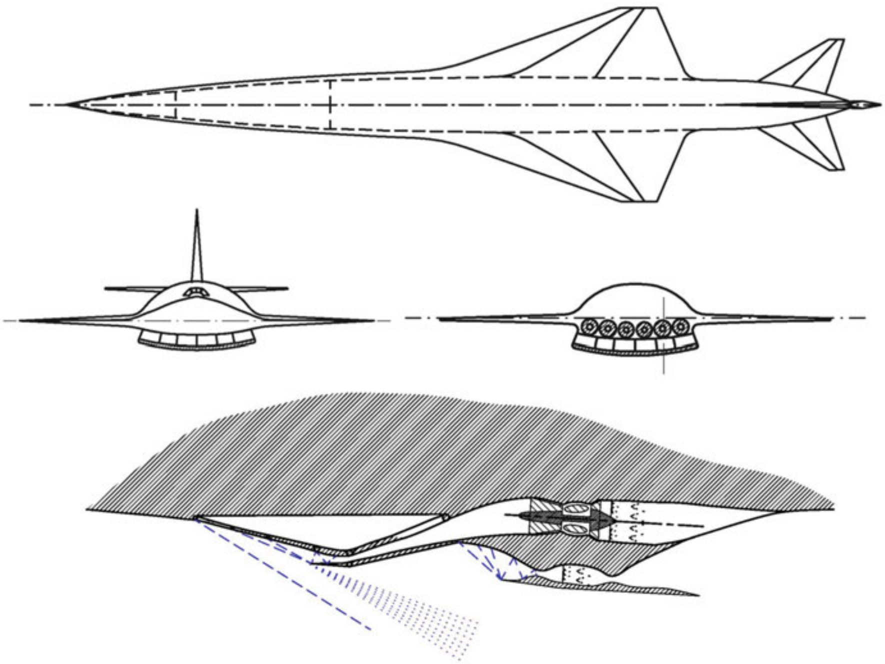


Fig. 6.41 Engine installed to fuselage HYCAT-1 and HYCAT-1A [34]

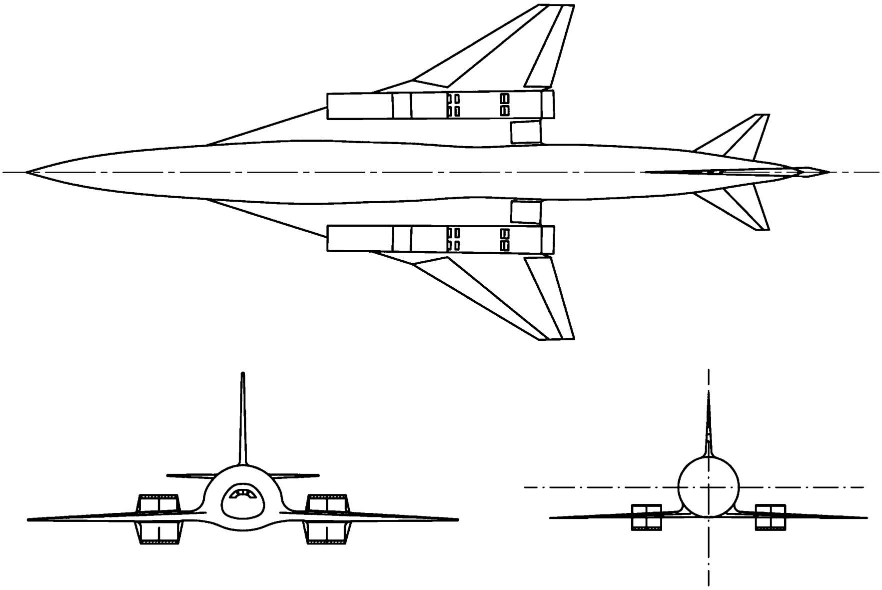


Fig. 6.42 Engine installed to the wing; HYCAT-4 configuration

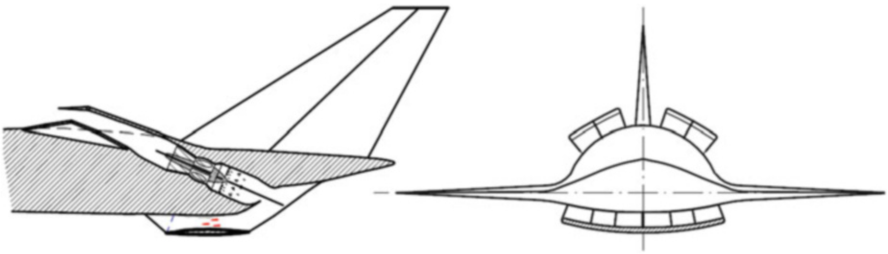


Fig. 6.43 Engine installed to the tail; HYCAT-2 [34]

6.4.5 Performance of TBCC (or Hybrid Engine)

As previously described, TBCC (or CCE) is composed of different engines performing different cycles, and each has strong and poor operation range [24] as illustrated in Fig. 6.44. Designer must then specifies which is which to be employed in each flight range and whether this engine will operate alone or in a combined mode with another engine.

Turbine-based engine (turbojet or turbofan) is used during takeoff, climb, as well as low supersonic Mach numbers. However, engine thrust will start to decrease at a certain Mach number depending on the overall pressure ratio of the engine and the bypass ratio for turbofans. In the same time ramjet generates thrust up to Mach numbers of 5–6 but generates zero or small thrust at takeoff or low subsonic speeds.

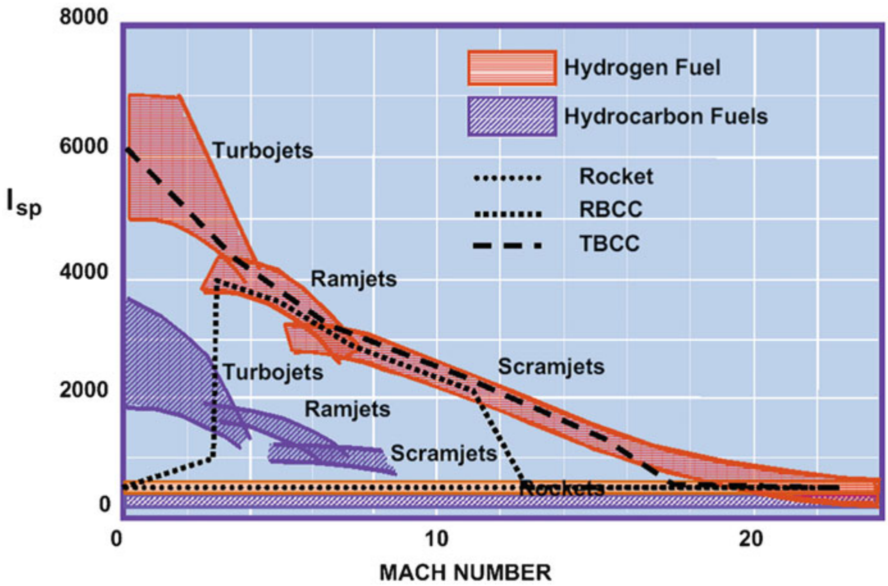


Fig. 6.44 Strong regimes for different propulsion cycles [25]

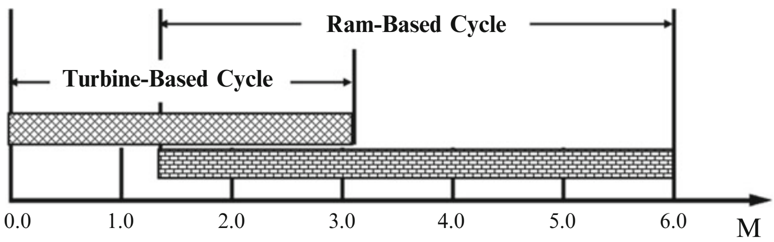


Fig. 6.45 The variation in TBCC with Mach number

As a result, designer set the turbojet/turbofan functioning at takeoff and operating alone up to say 1.5 Mach number. Next, turbojet (or turbofan) engine operates in conjunction with ramjet up to say a 3.0 Mach number. Finally, turbojet (or turbofan) engine is switched off leaving ramjet operating alone up to a Mach number of 6.0. Figure 6.45 shows cycle variation with Mach number for a Mach 6 civil airplane design [31].

Moreover, scramjet keeps generating thrust theoretically up to Mach number of 20. Thus, RBC operates in the scramjet mode for a Mach number range of 6.0–15.0. For space operation, this engine may have a booster rocket to help the engine in the transition region or drive the vehicle alone if it moves in space (the aerospace plane).

6.4.6 *Cycle Analysis of Turboramjet (or TBCC) Engine*

The analysis for both cases of wraparound and over–under turboramjet is given hereafter.

6.4.6.1 Wraparound Turboramjet

Designation of different states of engine is identified in Fig. 6.46. During low-speed flight, controllable flaps close a bypass duct and force air directly into the compressor section of the turbojet. During high-speed flight, the flaps block the flow into the turbojet, and the engine operates like a ramjet using the aft combustion chamber to produce thrust. In a combined mode, air may be split between both engines. Afterburner of turbojet acts as the combustion chamber of ramjet.

The engine cycle *has* two modes of operation, either working as a simple single-spool turbojet or as a ramjet. A plot for the cycle on the temperature, entropy plane, is illustrated in Fig. 6.47.

6.4.6.1.1 Operation as a Turbojet Engine

In the turbojet mode, a chain description of the different processes through the different modules is described in Table 6.5.

Different processes have been previously described in details. Input data for turbojet cycle will be: ambient pressure and temperature, flight Mach number, pressure ratio of compressor, turbine inlet temperature, maximum temperature in afterburner, type of nozzle, and lowest heating value for fuel used. Moreover, efficiencies of different modules (intake, compressor(s), combustion chamber,

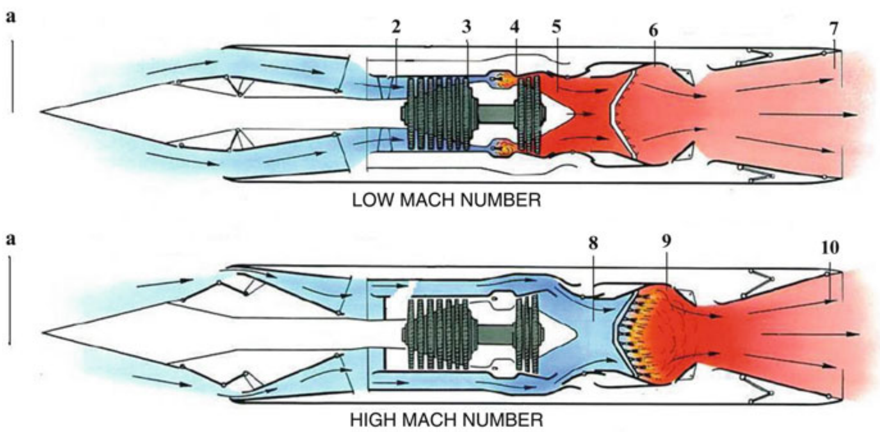


Fig. 6.46 Wraparound turboramjet

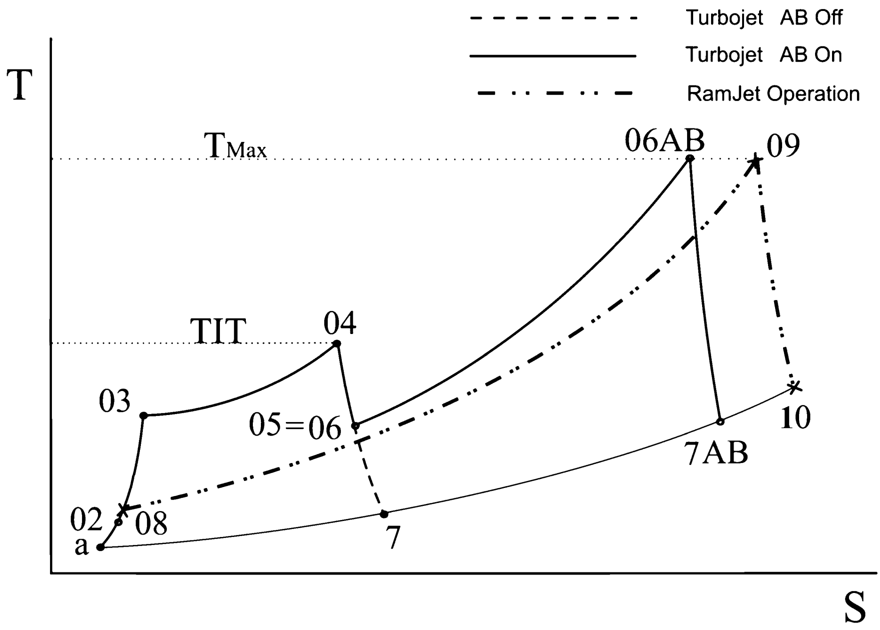


Fig. 6.47 T-s diagram of the wraparound turboramjet engine

Table 6.5 Modules of afterburning turbojet engine and corresponding processes

Part	States	Processes
Intake	a–2	Compression process with isentropic efficiency η_d
Compressor	2–3	Compression process with isentropic efficiency η_C
Combustion chamber	3–4	Heat addition at constant pressure process, with pressure drop $\Delta P_{c.c.}$ and burner efficiency η_b
Turbine	4–5	Expansion process with isentropic efficiency η_t
Afterburner	5–6	Heat addition at constant pressure process with pressure drop ΔP_{AB} and afterburner efficiency η_{ab}
Nozzle	6–7	Expansion process with isentropic efficiency η_N

turbine(s), afterburner and nozzle, mechanical efficiency) together with pressure drop in intake, combustion chamber, and afterburner. Calculations for temperature, pressure for different states, flight and exhaust speeds, as well as fuel burnt in combustion chamber and afterburner are given by Eqs. (6.1), (6.2), (6.3a), (6.3b), (6.4), (6.5), (6.6), (6.7a), (6.7b), (6.8), (6.9a), (6.9b), (6.9c), (6.9d), (6.10), (6.11a), (6.11b), (6.12), (6.13), (6.14), (6.15), (6.16), (6.17a), (6.17b), (6.18), (6.19a), (6.19b), and (6.20).

Single-spool turbojet engine is discussed here. However, the same procedure can be followed for double-spool turbojet engine.

Cycle performance parameters are calculated as described below:

The specific thrust is:

$$\left(\frac{T}{\dot{m}_a}\right)_{T.J} = (1 + f + f_{AB})V_7 - V \tag{6.72}$$

The thrust specific fuel consumption ($TSFC$):

$$(TSFC)_{T.J} = \frac{(f + f_{AB})}{T/\dot{m}_a} \tag{6.73}$$

The engine propulsive, thermal, and overall efficiencies are:

$$\eta_{\mathrm{p}} = \frac{\frac{T}{\dot{m}_a}V}{\frac{T}{\dot{m}_a}V + \frac{(V_7-V)^2}{2}(1 + f + f_{AB})} \tag{6.74}$$

$$\eta_{\mathrm{th}} = \frac{\frac{T}{\dot{m}_a}V + \frac{(V_7-V)^2}{2}(1 + f + f_{AB})}{Q_{HV}(f + f_{AB})} \tag{6.75}$$

$$\eta_{\mathrm{o}} = \eta_{\mathrm{p}} \times \eta_{\mathrm{th}} \tag{6.76}$$

6.4.6.1.2 Operation as a Ramjet Engine

In the ramjet mode, the flow is passing through the processes and states described in Table 6.6.

Different processes have been previously described in details. Input data for ramjet cycle will be: ambient pressure and temperature, flight Mach number, maximum temperature in combustion chamber, type of nozzle, and lowest heating value for fuel used. Moreover, efficiencies of different modules (intake, combustion chamber, and nozzle) together with pressure drop in intake and combustion chamber. Calculations for temperature, pressure for different states, flight and exhaust speeds, as well as fuel burnt in combustion chamber are given by Eqs. 5.19, 5.20, 5.21, 5.22, 5.23, 5.24, 5.25, and 5.26.

Ram cycle performance parameters will be calculated in the same way as in Chap. 5 and given here for the present states.

The thrust per unit air mass flow $\frac{T}{\dot{m}_a}$:

Table 6.6 Different modules of ramjet engine and the corresponding processes

Part	States	Processes
Intake	a–8	Compression process with total pressure ratio r_d
Combustion chamber	8–9	Heat addition at constant pressure with pressure drop $\Delta P_{c.c.}$
Nozzle	9–10	Expansion with isentropic efficiency (η_N).

$$\left(\frac{T}{\dot{m}_a}\right)_{RJ} = (1 + f_R)V_{10} - V \tag{6.77}$$

The thrust specific fuel consumption (*TSFC*):

$$(TSFC)_{RJ} = \frac{f_R}{\left(\frac{T}{\dot{m}_a}\right)_{RJ}} \tag{6.78}$$

The engine efficiencies propulsive, thermal, and overall efficiency:

$$\eta_{\text{p}} = \frac{\left(\frac{T}{\dot{m}_a}\right)_{RJ} V}{\left(\frac{T}{\dot{m}_a}\right)_{RJ} V + \frac{(V_{10}-V)^2}{2}(1 + f_R)} \tag{6.79}$$

$$\eta_{\text{th}} = \frac{\left(\frac{T}{\dot{m}_a}\right)_{RJ} V + \frac{(V_{10}-V)^2}{2}(1 + f_R)}{Q_{HV} f_R} \tag{6.80}$$

$$\eta_{\text{o}} = \eta_{\text{p}} \times \eta_{\text{th}} \tag{6.81}$$

6.4.6.1.3 Operation as a CCE or Dual Mode

Both turbojet and ramjet are operating simultaneously. The total thrust will be:

$$T = (T)_{TJ} + (T)_{RJ} \tag{6.82}$$

$$T = (\dot{m}_a)_{TJ}[(1 + f + f_{AB})V_7 - V] + (\dot{m}_a)_{RJ}[(1 + f_R)V_{10} - V] \tag{6.83a}$$

$$T = (\dot{m}_a)_{TJ}(1 + f + f_{AB})V_7 + (\dot{m}_a)_{RJ}(1 + f_R)V_{10} - (\dot{m}_a)_{TOTAL}V \tag{6.83b}$$

The specific thrust is:

$$\frac{T}{\dot{m}_a} = \frac{T_{\text{total}}}{(\dot{m}_a)_{\text{total}}} = \frac{(T)_{TJ} + (T)_{RJ}}{(\dot{m}_a)_{TJ} + (\dot{m}_a)_{RJ}} \tag{6.84}$$

The total fuel mass flow rate is:

$$\dot{m}_f = (\dot{m}_a)_{TJ}(f + f_{AB}) + (\dot{m}_a)_{RJ} \times f_R \tag{6.85}$$

The specific thrust fuel consumption is:

$$\frac{(\dot{m}_f)_{\text{Total}}}{(T)_{\text{Total}}} = \frac{(\dot{m}_f)_{TJ} + (\dot{m}_f)_{RJ}}{(T)_{TJ} + (T)_{RJ}} \tag{6.86}$$

6.4.6.2 Over–Under Turboramjet

For the over–under configuration of turboramjet engine, the engine height is large compared to the wraparound configuration. The reason is clear as the height in this case is the sum of the heights of turbojet and ramjet engines. Usually a part of the engine is to be buried inside the fuselage or inside the wing. Since the engine will operate at high Mach number, it needs a long intake. In the fuselage installation method, the vehicle underbody is completely integrated with the propulsion system as the upstream lower surface serves as an external compression system intake and the downstream surface as an external expansion nozzle (Fig. 6.50).

Three possible configurations for intake may be seen for different flight regimes (Fig. 6.49). In the first case at takeoff and low subsonic flight, the turbojet engine operates alone where a movable inlet ramp is deployed to allow for the maximum airflow rate into its intake and negligible or zero airflow rate into the ramjet engine (Fig. 6.49a). In the second case of higher Mach number, the engine operates at a dual mode where both turbojet and ramjet engines are operative for few seconds until Mach number reaches 2.5 or 3.0. In this case, moveable surfaces allow airflow rates into both engines (Fig. 6.49b). Finally, in the third case of hypersonic speed, the turbojet engine is shut down and only ramjet becomes operative (Fig. 6.49c). An example for such layout is found in the four over–under air-breathing turboramjet engines powering the Mach 5 WaveRider [32].

The complexity of both aerodynamic and mechanical design of two variable throat nozzles and necessary flaps led to another design including only one nozzle [33]. The suggested nozzle uses a single-expansion ramp nozzle (SERN) instead of a conventional, two-dimensional, convergent–divergent nozzle. Figures 6.50 and 6.51 illustrate a layout and T-s diagram of the over–under configuration of the turboramjet engine.

An over–under configuration has also three modes of operation, namely, a turbojet, a ramjet, or a dual mode. Dual mode represents the combined or transition mode in which both of the turbojet and ramjet are operating simultaneously.

6.4.6.2.1 Turbojet Mode

In this mode the engine is working as a simple turbojet engine and develops all the thrust needed by the aircraft. The states and governing equations are the same as the turbojet in the wraparound configuration.

6.4.6.2.2 Dual Mode

Both of the turbojet and ramjet are operating simultaneously. Turbojet starts declining and its developed thrust will be intentionally decreased by reducing the inlet air mass flow rate via its variable geometrical inlet. The ramjet starts working by adding fuel and starting ignition into its combustion chamber. The thrust generated in the ramjet is increased by increasing also the air mass flow rate through its variable area inlet. The generated thrust force will be then the sum of both thrusts of turbojet and ramjet engines. Same equations for wraparound case are to be applied here.

6.4.6.2.3 Ramjet Mode

In this mode, turbojet stops working and its intake is completely closed. All the air mass flow passes through the ramjet intake. With a Mach number increase, the forebody acts as a part of the intake with the foremost oblique shock wave located close to the aircraft nose, as shown in Fig. 6.48. Also same equations for wrap-around case will be employed here.

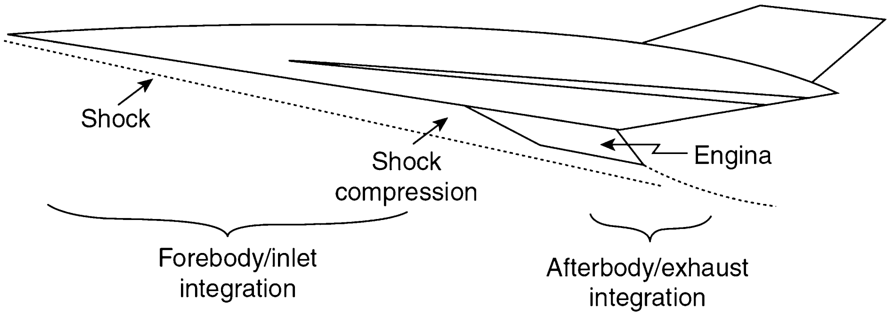


Fig. 6.48 Propulsion system integration with the vehicle body

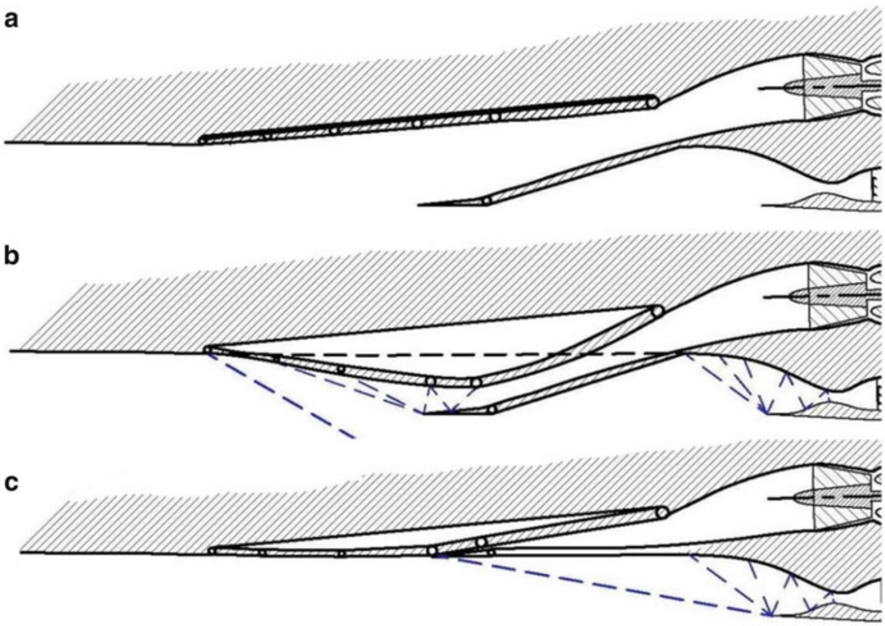


Fig. 6.49 Intake layout with different positions of intake flap at (**a**) subsonic, (**b**) low supersonic, and (**c**) hypersonic speeds [32]

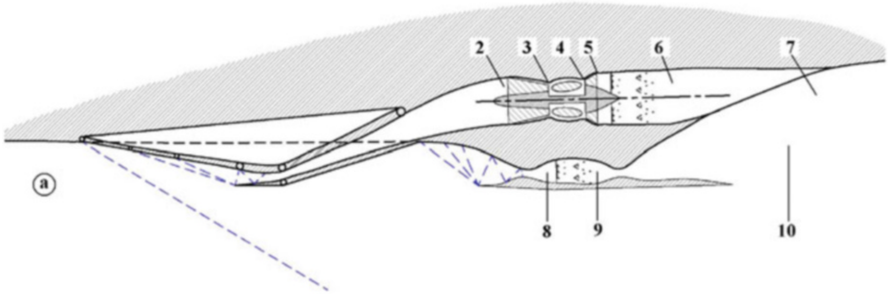


Fig. 6.50 Over–under layout of turboramjet engine

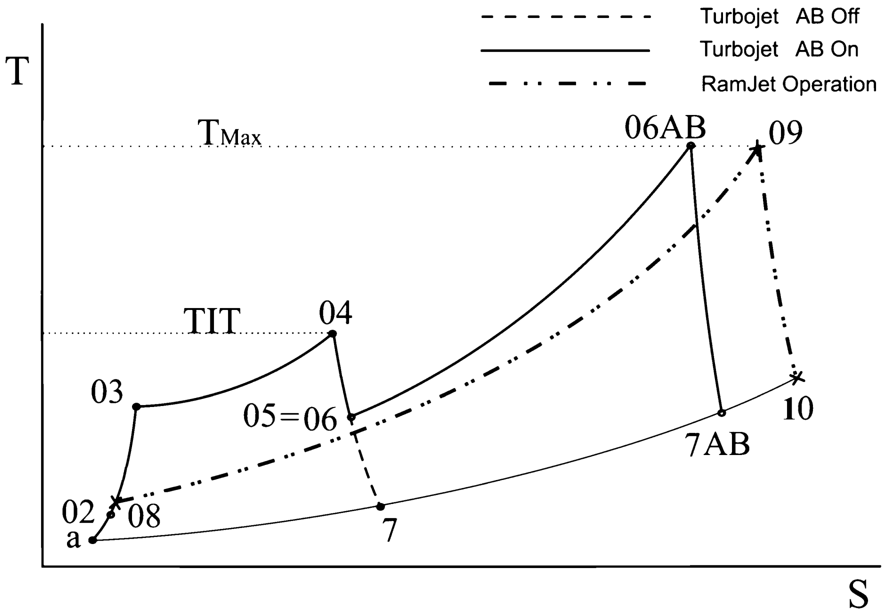


Fig. 6.51 T-s diagram of the over–under layout of turboramjet engine

6.4.7 General Analysis for a Turboramjet Engine

The turbine engine for a turboramjet may be either a turbojet or a of turbofan engine. In the entire previous sections, only turbojet engine was discussed. However, if turbofan engine replaces the turbojet engine, then additional parameters have to be specified, namely, fan pressure ratio and its isentropic efficiency besides bypass ratio together with the other parameters listed in Sect. 6.4.6.1.1. Moreover,

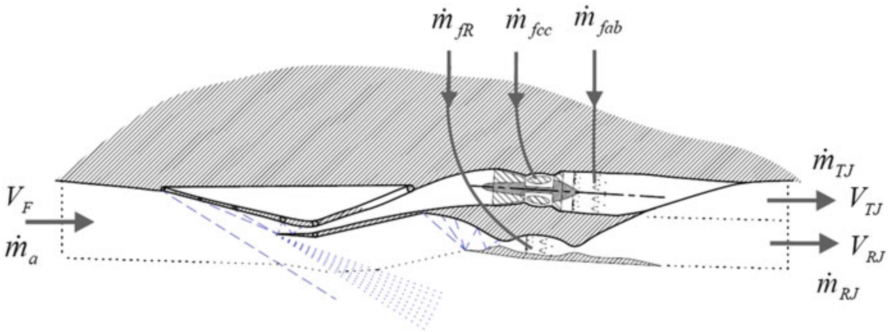


Fig. 6.52 Turboramjet engine and appropriate control volume

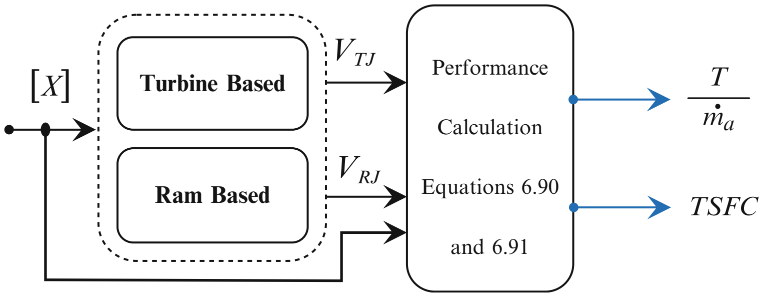


Fig. 6.53 Block diagram for CCE

for high supersonic speed, the ram-based engine will be a scramjet rather than a ramjet engine. Figure 6.52 shows the control volume for the general turbo ramjet engine. Figure 6.53 shows a block digram of the complete system.

The air mass flow through the turbine-based cycle is $\dot{m}_{aTJ}$, while the air mass flow through the ram-based cycle is $\dot{m}_{aRJ}$. From cycle analysis the fuel mass flow rates will be calculated. Apply the mass conservation law (continuity equations for both cycle to reach Eqs. (6.87) and (6.88). Momentum equation defines the thrust force for a full expansion of gases in nozzle(s), Eq. (6.89). The specific thrust and *TSFC* can be obtained through Eqs. (6.90) and (6.91):

$$\dot{m}_{TJ} = \frac{\dot{m}_{aTJ}}{\dot{m}_a}\dot{m}_a + \dot{m}_{fcc} + \dot{m}_{fab} \tag{6.87}$$

$$\dot{m}_{RJ} = \frac{\dot{m}_{aRJ}}{\dot{m}_a}\dot{m}_a + \dot{m}_{fR} \tag{6.88}$$

$$T = \dot{m}_{aTJ} V_{TJ} + \dot{m}_{aRJ} V_{RJ} - \dot{m}_a V_F \tag{6.89}$$

$$\frac{T}{\dot{m}_a} = \left(\frac{\dot{m}_{aTJ}}{\dot{m}_a} + \frac{\dot{m}_{fcc}}{\dot{m}_a} + \frac{\dot{m}_{fab}}{\dot{m}_a} \right) V_{TJ} + \left(\frac{\dot{m}_{aRJ}}{\dot{m}_a} + \frac{\dot{m}_{fR}}{\dot{m}_a} \right) V_{RJ} - V_F \tag{6.90}$$

$$TSFC = \frac{\dot{m}_{fcc}/\dot{m}_a + \dot{m}_{fab}/\dot{m}_a + \dot{m}_{fR}/\dot{m}_a}{T/\dot{m}_a} \tag{6.91}$$

Example 6.12 A conceptual design of a turboramjet engine to power a future supersonic transport; Fig. (6.54), provided the following data:

Flight altitude	20,000 m
Cruise Mach number	3.0
Engines	Two turboramjet engines in over–under installation with separate intakes and nozzles
Hydrogen fuel with heating value	119 MJ/kg

Turbojet engine

Single-spool afterburning

Intake efficiency	η_I	92 %
Compressor pressure ratio	π_C	7
Compressor efficiency	η_C	92 %
Burner efficiency	η_{CC}	99 %
CC pressure ratio	π_{CC}	0.975
Turbine inlet temperature	TIT = 1500 K	
Turbine efficiency	η_T	92 %
Mechanical efficiency	η_m	99 %
Afterburner maximum temperature = 1750 K		
Afterburner efficiency	η_{Ab}	99 %
Afterburner pressure ratio	π_{Ab}	0.98
Nozzle efficiency	η_N	99 %

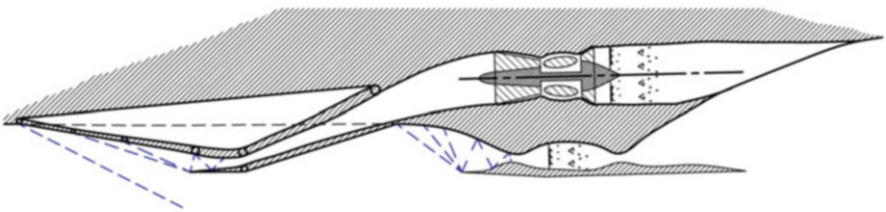


Fig. 6.54 Turboramjet engine

Ramjet engine

Ramjet intake efficiency	η_{RI}	95 %
Ram burner maximum temperature	=	1900 K
Ram burner efficiency	η_{Rb}	99 %
Ram burner pressure ratio	π_{Rb}	0.98
Ramjet nozzle efficiency	η_{RN}	99 %

The ratio between air mass flow rate in ramjet and turbojet engines is $\dot{m}_{aRJ}/\dot{m}_{aTJ} = \alpha_R = 2$.

1. For the combined engine (TBCC), *calculate*:
   (a) Specific thrust
   (b) Thrust specific fuel consumption
   (c) Thermal efficiency
   (d) Propulsive efficiency
   (e) Overall efficiency
2. Compare the above results with the cases:
   (a) The turbojet engine is operating alone.
   (b) The ramjet engine is operating alone.
3. If the total thrust generated is 150 kN, calculate:
   (a) Air mass flow rate through each engine
   (b) Thrust generated by each engine

Solution
Ambient conditions at 20,000 m altitude are:

$$T_a = 216.66 \text{ k} \quad P_a = 5{,}475.5 \text{ Pa}$$

Then at flight Mach number $M_1 = 3$.

Turbojet

Layout of afterburning turbojet and the corresponding T-s diagram are illustrated by Fig. (6.55).

(a) *Intake 1–2*

$$T_{02} = T_{01} = T_a\left(1 + \frac{\gamma_c - 1}{2}M_1^2\right) = 216.66\left(1 + \frac{0.4}{2} \times 9\right) = 606.65 \text{ K}$$

$P_{02} = P_{01}$

$$= P_a\left(1 + \eta_I\frac{\gamma_c - 1}{2}M_1^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} = 5475.5\left(1 + 0.92 \times \frac{0.4}{2} \times 9\right)^{\frac{1.4}{0.4}} = 167{,}190 \text{ Pa}$$

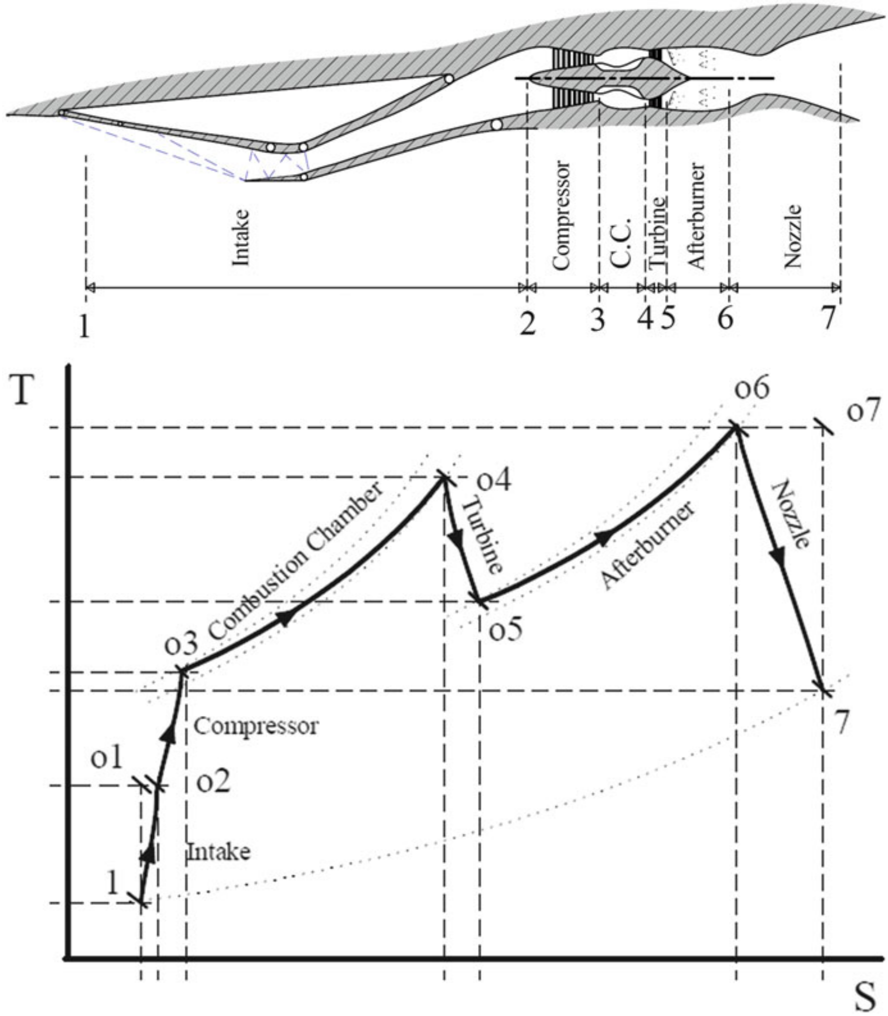


Fig. 6.55 Stations and T-s diagram of turboramjet engine

(b) *Compressor 2–3*

$$P_{03} = P_{02}\pi_C = 1170400 \text{ Pa}$$

$$T_{03} = T_{02}\left[1 + \frac{1}{\eta_C}\left(\pi_C^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)\right] = 606.65\left[1 + \frac{1}{0.92}\left(7^{\frac{0.4}{1.4}} - 1\right)\right] = 1097 \text{ K}$$

(c) *Combustion chamber 3–4*

$$T_{04} = TIT = 1500 \text{ K}$$

$$P_{04} = P_{03}\pi_{CC} = 1141,100 \text{ Pa}$$

$$f = \frac{C_{Ph}T_{04} - C_{Pc}T_{03}}{\eta_{cc}Q_{HV} - C_{Ph}T_{04}} = \frac{1148 \times 1500 \ - 1005 \times 1097}{0.99 \times 119 \times 10^6 - 1148 \times 1500} = 5.337 \times 10^{-3}$$

(d) *Turbine 4–5*

$$T_{o5} = T_{o4} - \frac{C_{Pc}(T_{o3} - T_{o2})}{\eta_m C_{Ph}(1+f)} = 1500 - \frac{1005(1097 - 606.65)}{0.99 \times 1148(1.005337)} = 1067.85 \ \text{K}$$

$$P_{05} = P_{04}\left[1 - \frac{1}{\eta_t}\left(1 - \frac{T_{05}}{T_{04}}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}} = 1,141,100\left[1 - \frac{1}{0.92}\left(1 - \frac{1067.85}{1500}\right)\right]^{\frac{4/3}{1/3}} = 253,960 \ \text{Pa}$$

(e) *Afterburner 5–6*

$$T_{06} = T_{\text{Max } Tj} = \ 1750 \ \text{K}$$

$$P_{06} = P_{05}\pi_{Ab} = 248,880 \ \text{Pa}$$

$$f_{Ab} = \frac{C_{Ph}(T_{06} - T_{05})(1+f)}{\eta_{Ab}Q_{HV} - C_{Ph}T_{06}(1+f)} = \frac{1148(1750 - 1067.85) \ \times 1.00534}{0.98 \times 10^6 - 1148 \times 1750 \times 1.00534}$$
$$= \ 6.87 \times 10^{-3}$$

(f) *Nozzle 6–7*

$$P_7 = P_1 = 5475.5 \ \text{Pa}$$

$$T_7 = T_{06}\left[1 - \eta_N\left(1 - \left(\frac{P_7}{P_{06}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right)\right] = 1750\left[1 - 0.99\left(1 - \left(\frac{5475.5}{248,880}\right)^{\frac{1/3}{4/3}}\right)\right]$$
$$= 684.74 \ \text{K}$$

$$V_{TJ} = V_7 = \sqrt{2C_{Ph}(T_{06} - T_7)} = \sqrt{2 \times 1148(1750 - 684.74)} = 1563.9 \ \text{m/s}$$

Ramjet

Figure (6.56) illustrates the states of a ramjet engine while its T-s diagram is displayed in Fig. (6.57).

(a) *Intake 1r–2r*

$$T_{02r} = T_{01r} = T_a\left(1 + \frac{\gamma_c - 1}{2}M_1^2\right) = 216.66\left(1 + \frac{0.4}{2} \times 9\right) = 606.65 \ \text{K}$$

$$P_{02r} = P_{01r} = P_a\left(1 + \eta_{RI}\tfrac{\gamma_c - 1}{2}M_1^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} = 5475.5\left(1 + 0.95 \times \tfrac{0.4}{2} \times 9\right)^{\frac{1.4}{0.4}}$$
$$= 179,400 \ \text{Pa}$$

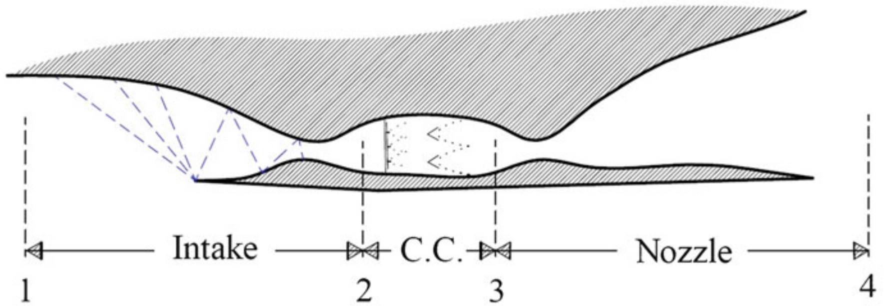


Fig. 6.56 Ramjet operation

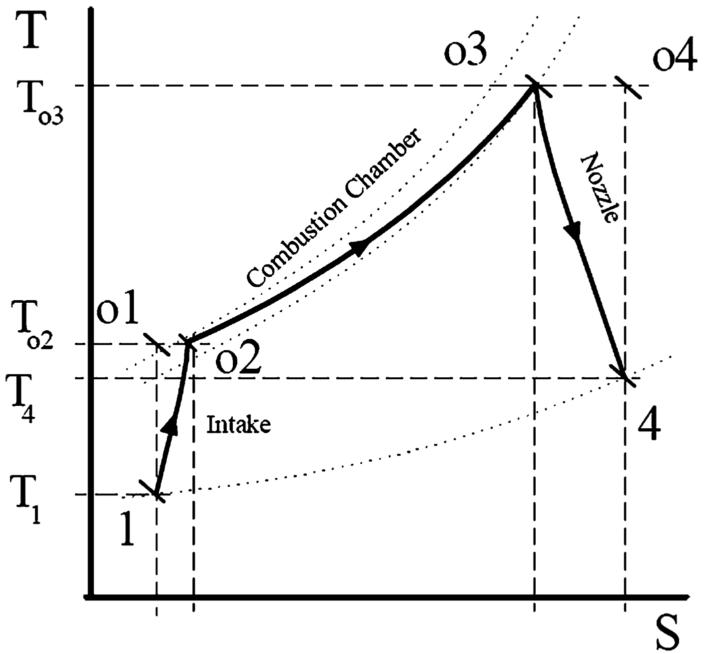


Fig. 6.57 T-s diagram of ramjet operation

(b) *Combustion chamber 2r–3r*

$$T_{03r} = T_{\mathrm{Max}\ Rj} = 1900\ \ \mathrm{K}$$

$$P_{03r} = P_{02r}\pi_{Rb} = 175,810\ \ \mathrm{Pa}$$

$$f_{Rb} = \frac{C_{Ph}T_{03r} - C_{Pc}T_{02r}}{\eta_{Rb}Q_{HV} - C_{Ph}T_{03r}}$$

$$f_{Rb} = \frac{1148*1900 - 1005*606.65}{0.99*119 \times 10^6 - 1148*1900}$$
$$f_{Rb} = 13.59 \times 10^{-3}$$

(c) *Nozzle 3r–4r*

$$P_{4r} = P_{1r} = 5,475.5 \text{ Pa}$$
$$T_{4r} = T_{03r}\left[1 - \eta_{RN}\left(1 - \left(\frac{P_{4r}}{P_{03r}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right)\right]$$
$$= 1900\left[1 - 0.99\left(1 - \left(\frac{5475.5}{1,75,810}\right)^{\frac{1/3}{4/3}}\right)\right] = 809.2 \text{ K}$$
$$V_{RJ} = V_4 = \sqrt{2C_{Ph}(T_{03} - T_4)} = \sqrt{2 \times 1148(1900 - 809.2)} = 1582.6 \text{ m/s}$$

Performance

$$V_1 = M\sqrt{\gamma_c RT_1} = M\sqrt{\gamma_c RT_1} = 885.15 \text{ m/s}$$
$$\dot{m}_{fTJ} = \dot{m}_{aTJ}(f + f_{Ab})$$
$$\dot{m}_{fRJ} = \dot{m}_{aRJ} f_{Rb}$$
$$\dot{m}_f = \dot{m}_{fRJ} + \dot{m}_{fTJ}$$
$$\dot{m}_a = \dot{m}_{aTJ} + \dot{m}_{aRJ} = \dot{m}_{aTJ}(1 + \alpha_R)$$
$$T = \left(\dot{m}_{aTJ} + \dot{m}_{fTJ}\right)V_{TJ} + \left(\dot{m}_{aRJ} + \dot{m}_{fRJ}\right)V_{RJ} - \dot{m}_a V_1$$
$$T = \dot{m}_{aTJ}(1 + f + f_{Ab})V_{TJ} + \dot{m}_{aRJ}(1 + f_{Rb})V_{RJ} - (\dot{m}_{aTJ} + \dot{m}_{aRJ})V_1$$
$$T/\dot{m}_a = \frac{(1 + f + f_{Ab})V_{TJ}}{[1 + \alpha_R]} + \frac{\alpha_R(1 + f_{Rb})V_{RJ}}{[1 + \alpha_R]} - V_1 = 711.7 \text{ N.s/kg}$$
$$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_{aTJ}(f + f_{Ab}) + f_{Rb}\dot{m}_{aRJ}}{\dot{m}_{aTJ}(1 + f + f_{Ab})V_{TJ} + \dot{m}_{aRJ}(1 + f_{Rb})V_{RJ} - (\dot{m}_{aTJ} + \dot{m}_{aRJ})V_1}$$
$$TSFC = \frac{f + f_{Ab} + \alpha_R f_{Rb}}{(1 + f + f_{Ab})V_{TJ} + \alpha_R(1 + f_{Rb})V_{RJ} - (1 + \alpha_R)V_1}$$
$$= 1.84556 \times 10^{-5} \text{ kg.s/N}$$
$$\eta_{th} = \frac{\Delta K.E}{\dot{Q}_{add}} = \frac{\dot{m}_{aTJ}(1 + f + f_{Ab})V_{TJ}{}^2 + \dot{m}_{aRJ}(1 + f_{Rb})V_{RJ}{}^2 - (\dot{m}_{aTJ} + \dot{m}_{aRJ})V_1{}^2}{2Q_{HV}(\dot{m}_{aTJ}*f*\eta_{cc} + \dot{m}_{aTJ}*f_{Ab}*\eta_{Ab} + \dot{m}_{aRJ}*f_{Rb}*\eta_{Rb})}$$
$$\eta_{th} = \frac{(1 + f + f_{Ab})V_{TJ}{}^2 + \alpha_R(1 + f_{Rb})V_{RJ}{}^2 - (1 + \alpha_R)V_1{}^2}{2Q_{HV}(f*\eta_{cc} + f_{Ab}*\eta_{Ab} + \alpha_R*f_{Rb}*\eta_{Rb})} = \frac{5,200,314.3}{9,267,986.6}$$
$$= 56.11\%$$

$$\eta_{pr} = \frac{T^*V_1}{\Delta K.E}$$
$$= \frac{2V_1(\dot{m}_{aTJ}(1+f+f_{Ab})V_{TJ} + \dot{m}_{aRJ}(1+f_{Rb})V_{RJ} - (\dot{m}_{aTJ} + \dot{m}_{aRJ})V_1)}{\dot{m}_{aTJ}(1+f+f_{Ab})V_{TJ}{}^2 + \dot{m}_{aRJ}(1+f_{Rb})V_{RJ}{}^2 - (\dot{m}_{aTJ} + \dot{m}_{aRJ})V_1{}^2}$$
$$\eta_{pr} = \frac{2V_1((1+f+f_{Ab})V_{TJ} + \alpha_R(1+f_{Rb})V_{RJ} - (1+\alpha_R)V_1)}{(1+f+f_{Ab})V_{TJ}{}^2 + \alpha_R(1+f_{Rb})V_{RJ}{}^2 - (1+\alpha_R)V_1{}^2} = \frac{3,779,721}{5,200,314.3}$$
$$= 72.68\ \%$$
$$\eta_o = \frac{T^*V_1}{\dot{Q}_{\text{add}}} = \eta_{pr}{}^*\eta_{th} = 40.78\ \%$$

	Turbojet only	*Ramjet only*	*TBCC engine*
$T/\dot{m}_a$(N.s/kg)	697.15	718.95	711.7
TSFC (kg/N.s)	1.7534 E^{-5}	1.89 E^{-5}	1.84556 E^{-5}
η_{th}	59 %	54.81 %	56.11 %
η_{pr}	73.03 %	72.52 %	72.68 %
η_o	43.09 %	39.75 %	40.78 %

(C) From the above table:

Total thrust: $T = \dot{m}_{aTJ} \times (T/\dot{m}_a)_{TJ} + \dot{m}_{aRJ} \times (T/\dot{m}_a)_{RJ}$

With $\dot{m}_{aRJ}/\dot{m}_{aTJ} = \alpha_R = 2$

Then $T = \dot{m}_{aTJ} \times \left[(T/\dot{m}_a)_{TJ} + 2 \times (T/\dot{m}_a)_{RJ}\right]$

$$150,000 = \dot{m}_{aTJ} \times [697.15 + 2 \times 718.95]$$

Air mass flow rate through turbojet engine	$\dot{m}_{aTJ} = 70.26\ \text{kg/s}$
Air mass flow rate through ramjet engine	$\dot{m}_{aRJ} = 140.52\ \text{kg/s}$
Thrust generated by turbojet engine	$T_{TJ} = \dot{m}_{aTJ} \times (T/\dot{m}_a)_{TJ} = 48,974\ \text{N}$
Thrust generated by ramjet engine	$T_{RJ} = \dot{m}_{aRJ} \times (T/\dot{m}_a)_{RJ} = 101,026\ \text{N}$

Example 6.13 The turboramjet engine discussed in Example (6.12) will be used in propelling the same aircraft during its operation from flight Mach numbers of 2–4 at same 20,000 m altitude. For Mach number 2, only the turbojet engine will be used while the ramjet is off. At Mach number 4, the ramjet will be only operating while the turbojet will be off.

The drag coefficient for aircraft at different Mach numbers is given in Table 6.7.

Assuming aircraft is moving at constant speeds where thrust and drag forces are equal, calculate:

1. Drag force at Mach numbers of 2 and 4
2. Air mass flow rate for both turbojet and ramjet engines

Table 6.7 Drag coefficient vs. Mach number

M	2.0	3.0	4.0
C_D	0.0569	0.038	0.026

Solution

1. *At Mach number = 2*

Turbojet engine with lit afterburner is only operating.

Same procedure will be followed as in the previous example. Only final results will be given here:

Module	Intake	Compressor	CC	Turbine	AB	Nozzle
Outlet temperature (K)	390	705.23	1500	1196	1750	870.2
Outlet pressure (Pa)	37,744	264,208	257,603	95,210	93,306	5475.5
Fuel-to-air ratio	–	–	0.008726	–	0.005598	–
Velocity (m/s)	590.1	–	–	–	–	1421.3

The thrust force

$$\begin{aligned} T &= \dot{m}_{aTJ}[(1+f+f_{Ab})V_{TJ} - V_1] \\ &= \dot{m}_{aTJ}[(1+0.008726+0.005598)\times 1421.3 - 590.1] \end{aligned}$$

$$T = 851.56\ \dot{m}_{aTJ}$$

$$\text{Since} \qquad T = D = 0.5\rho V_1^2 A C_D = 0.5\rho M_1^2 a_1^2 A C_D$$

$$\frac{T_{TJ}}{T_{\text{combined}}} = \left(\frac{M_{TJ}}{M_{\text{combined}}}\right)^2 \frac{(C_D)_{TJ}}{(C_D)_{\text{combined}}} = \left(\frac{2}{3}\right)^2 \times \frac{0.0569}{0.038} = 0.6655$$

Drag and thrust forces are $\quad D_{TJ} = T_{TJ} = 0.6655 \times 150,000 = 99,825\ \text{N}.$

$$\text{But} \qquad T = 851.56\,\dot{m}_{aTJ}$$

Air mass flow rate through turbojet engine is $\quad \dot{m}_{aTJ} = \frac{99,825}{851.56} = 117.225\ \text{kg/s}.$

2. *At Mach number = 4*

Ramjet engine is only operating.

Same procedure will be followed as in the previous example. Only final results will be given here:

Module	Intake	CC	Nozzle
Outlet temperature (K)	910	1900	588
Outlet pressure (kPa)	667.118	653.776	5.475
Fuel-to-air ratio	–	0.01095	–
Speed (m/s)	1180	–	1736

$$T = \dot{m}_{aRJ}[(1 + f_{RJ})V_{RJ} - V_1] = \dot{m}_{aRJ}[(1 + 0.01095) \times 1736 - 1180]$$
$$= 574.6\ \dot{m}_{aRJ}$$

$$T = 574.6\ \dot{m}_{aRJ}$$

$$T = D = 0.5\rho V_1^2 A C_D = 0.5\rho M_1^2 a_1^2 A C_D$$

$$\frac{T_{RJ}}{T_{combined}} = \left(\frac{M_{RJ}}{M_{combined}}\right)^2 \frac{(C_D)_{RJ}}{(C_D)_{combined}} = \left(\frac{4}{3}\right)^2 \times \frac{0.026}{0.038} = 1.216374$$

Drag and thrust forces are $D_{RJ} = T_{RJ} = 1.216374 \times 150{,}000 = 182{,}456\,N$.

$$\text{But} \qquad T_{RJ} = 574.6\ \dot{m}_{aRJ}$$

Air mass flow rate through ramjet engine is $\dot{m}_{aRJ} = \frac{182{,}456}{574.6} = 317.53\ \text{kg/s}$.

6.4.8 Design Procedure

A simplified design procedure for a turboramjet engine will be given here [35–37]. The following steps summarize the procedure for high supersonic/hypersonic cruise aircraft propulsion integration:

1. Mission or flight envelope has to be selected. The cruise altitude is important from fuel economy point of view, while takeoff, climb, and acceleration are critical from the surplus thrust that must be available over the aircraft drag force.
2. Calculate the drag force based on drag coefficient variations with Mach number.
3. The needed thrust force must be greater than the drag value all over its Mach number operation. A margin of 10 % is selected here.
4. Determination of the number of engines based on the maximum thrust suggested for the hybrid engine at its different modes.
5. The performance of each constituents of the hybrid engine is separately determined. For example, if the hybrid engine includes a ramjet, turbojet, and scramjet, each module is examined separately. Thus, the performance of turbojet engine running alone is considered first. The specific thrust and thrust specific fuel consumption are determined for the cases of operative and inoperative afterburner. Next, the performance of the ramjet operating alone also is considered. Since the ramjet operates up to Mach number of 6, liquid hydrogen is considered as its fuel. If a scramjet is available, then its performance is determined separately also.
6. Optimization of the hybrid engine represents the next difficult step. From the specific thrust and thrust specific fuel consumption, the switching points are defined. Thus, in this step, the designer selects at which Mach number the turbojet operates or stops the afterburner and, moreover, at which Mach number

the turbojet must be completely stopped and replaced by the ramjet. If a scramjet is also available, another decision has to be made concerning the switching Mach number from ramjet to scramjet. The switching process duration and the accompanying procedures regarding the air and fuel flow rates and inlet doors actuation are to be carefully defined.

6.4.9 Future TBCC Engine

The *Lockheed Martin SR-72* (Fig. 6.58) is a conceptualized unmanned, hypersonic aircraft intended for intelligence, surveillance and reconnaissance proposed by Lockheed Martin to succeed the retired Lockheed SR-71 Blackbird. The SR-72 was dubbed by *Aviation Week* as "son of Blackbird [38]". It could be operational by 2030. Moreover, it would fly at speeds up to Mach 6.

The SR-72 will use a turbine-based combined-cycle (TBCC) system. Thus, a turbine engine will be used from takeoff up to Mach 3, while a dual-mode ramjet engine will accelerate the vehicle to hypersonic speeds. The turbine and ramjet engines share common inlet that provides air to both turbine and ramjet and a common nozzle to discharge exhaust from both engines. Variable inlet and nozzle ramps open and close to match the cycle requirements.

6.5 Conclusion

This chapter is devoted to turbine-based cycle (TBC). Three major types of engines are analyzed, namely, turbojet, turbofan, and turboramjet engines. Application to both airliners and missiles is discussed. Detailed classifications and thermodynamic and performance analyses are discussed.

Problems

6.1 Complete the following table.

Engine	Description	Examples	Application	Advantages	Disadvantages
Turbojet					
Turbofan					
Propfan					
Turboprop					
Turboshaft					

6.2 A. What is the difference between a jet engine and a rocket engine?
B. Can a jet engine use hydrogen fuel instead of a fossil one? If so, is its structure or operation any different from the turbofan or turbojet?
Turbojet

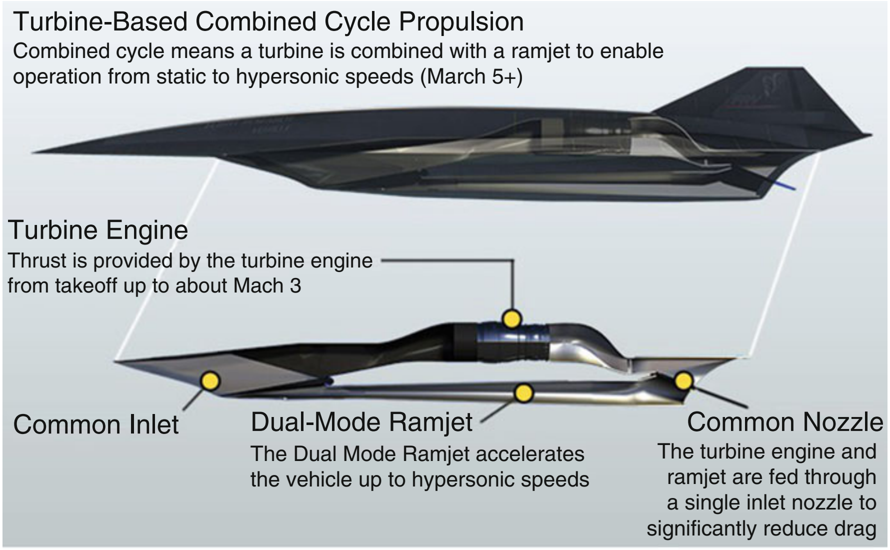


Fig. 6.58 Lockheed martin SR-72

6.3 Choose the correct answer:

(a) "Afterburning" in a jet engine involves burning additional fuel in the:

(A) Jet pipe
(B) Turbine
(C) Combustion chamber
(D) Compressor

(b) The function of the turbine in a turbojet engine is to:

(A) Vaporize the fuel as much as possible
(B) Drive the gas stream into the atmosphere
(C) Energize the gas steam
(D) Drive the compressor

(c) Which of these is a turbojet engine?

(A) Olympus 593
(B) Spey
(C) Tyne
(D) Pegasus

6.4 An aircraft equipped with a *turbojet engine* rolls down the runway at 180 km/h. The engine consumes air at 55 kg/s and produces an exhaust velocity of 160 m/s.

(a) What is the thrust of this engine?
(b) What are the magnitude and direction if the exhaust is deflected 60, 90, and 120° without affecting mass flow?

6.5 A turbojet engine propels an aircraft at 600 mph. The fuel heating value is 44 MJ/kg, the speed ratio $\frac{V_e}{V_\infty} = 2$, and the fuel-to-air ratio $f = 0.02$.
Determine:

(a) Propulsion efficiency
(b) Thermal efficiency
(c) Overall efficiency

6.6 Compare between the ideal performance of ramjet and turbojet engines in the following operating case:

- Flight Mach number for both is 1.5.
- Ambient conditions are $Ta = 216.7$, $P_a = 14.1$ kPa.
- Maximum total temperature: turbojet $= 1400$ K, ramjet $= 2500$ K.
- Compressor pressure ratio of turbojet $= 10$.

Assume

All processes are ideal
Unchoked nozzles
Fuel heating value $= 45$ MJ/kg
Constant properties ($\gamma = 14$, $C_p = 1.005$ kJ/(kg.K)

Compare the performance of both engines by calculating:

(a) *TSFC*
(b) Specific thrust
(c) Propulsive efficiency
(d) Thermal efficiency

If the turbojet engine is fitted with an afterburner that develops same maximum temperature as ramjet (2500 K), compute the above four requirements (a–d). Comment:

6.7 A twin-engine *turbojet* airplane flies at $M = 0.8$, $L/D = 5$, airplane weight $W = 15$ t.

Operating data:

$$P_a = 1 \text{ bar}, \ T_a = 288 \text{ K}, \ \gamma = 1.4, \ C_p = 1.005 \text{ kJ/kg.K}, Q_R = 43 \text{ MJ/kg}$$
$$TIT = 1400 \text{ K}, \ \pi_0 = 12, \ \eta_d = 0.9, \ \eta_c = 0.92, \ \eta_b = 0.99, \ \eta_t = 0.95$$
$$\eta_n = 0.96 \ , \ \eta_m = 0.99, \ \Delta P_{cc} = 4 \ \%,$$

Calculate

- The stagnation pressures and temperatures at each engine station
- The thrust per engine
- *TSFC*

If the afterburner of the jet engine is activated and the fuel flow is $\dot{m}_f = 1.63 \text{ kg/s}$ both in the combustor and the afterburner, *calculate* the increase in thrust and the change in *TSFC* because of the operating afterburner.

6.8 A General Electric J79-GE-15 *turbojet engine* is one of two engines propelling McDonnell F4C airplanes (wing area $S = 530$ ft^2, inlet area for each engine $A_I = 6.5$ ft^2) cruising at a constant Mach number $M_0 = 0.84$ at an altitude of 35,000 ft. The drag coefficient of the aircraft under these conditions is 0.044. Assume that the nozzle is unchoked and the turbine exit stagnation temperature is 990 K.

Determine

(a) Net thrust of the engine
(b) Gross thrust of the engine
(c) Weight (or mass) flow through the engine
(d) Exhaust velocity
(e) Exit static temperature
(f) Exit Mach number
(g) Propulsive, thermal, and overall efficiencies

6.9 The airplane SR-71 fitted with two Pratt & Whitney turbojet engine J58 that incorporates an afterburner. All processes are assumed ideal with constant values of both (C_p and γ).

(A) *Draw* a schematic diagram for the engine identifying each states by a number from (1–7).
(B) *Draw* the corresponding cycle on T-s and P-v diagrams.
(C) *Prove* that:

The *thrust ratio* for operative (τ_{AB}) and inoperative afterburner (τ) is expressed by the relation:

$$\frac{\tau_{AB}}{\tau}=\frac{\sqrt{\left(\frac{2}{\gamma-1}\right)\left(\frac{T_{06A}}{T_a}\right)\left[1-\left(\frac{P_a}{P_{06A}}\right)^{\frac{\gamma-1}{\gamma}}\right]}-M}{\sqrt{\left(\frac{2}{\gamma-1}\right)\left(\frac{T_{05}}{T_a}\right)\left[1-\left(\frac{P_a}{P_{05}}\right)^{\frac{\gamma-1}{\gamma}}\right]}-M}$$

where subscript (a) denotes ambient conditions and ($6A$) denotes inlet conditions to nozzle when the afterburner is operative.

Next, when $M=0$ and outlet temperatures of both compressor and turbine are equal, then:

$$\frac{\tau_{AB}}{\tau}=\frac{\sqrt{\left(\frac{T_{06A}}{T_a}\right)}}{\pi^{\left(\frac{\gamma-1}{2\gamma}\right)}}$$

6.10 The below table gives details of temperature and pressure at interior states of an afterburning turbojet engine

	Diffuser inlet	Compressor inlet	Compressor outlet	Turbine inlet	Afterburner inlet	Afterburner outlet	Nozzle outlet
T [K]	220	226	424.3	1300	1082	2200	1650
P [kPa]	23.1	25.1	188.25	182.5	82.5	79	23.1

Assuming *variable values* of both (C_p and γ), calculate:

(a) Efficiency of diffuser, compressor, and turbine
(b) Percentage of pressure drop in combustion chamber and afterburner
(c) Total rate of burnt fuel in engine if $Q_R=44\ \text{MJ/g}$, $\eta_{\text{burner}}=0.96$ and air mass flow rate = 135 kg/s
(d) The thrust force if the inlet area is 0.25 m^2

6.11 An aircraft powered by a turbojet engine is flying with 900 km/h. The engine takes 50 kg/s of air and expands the gases to the ambient pressure. The fuel-to-air ratio is 50 and heating value is 43 MJ/kg. For a maximum thrust condition ($u_f = 05u_e$), calculate:

(A) Jet velocity
(B) Thrust
(C) Specific thrust
(D) Propulsive and thermal efficiency
(E) *TSFC*

6.12 An aircraft powered by a turbojet engine is flying with 280 m/s at an altitude of 25 km, where the ambient pressure and temperature are 25.11 kPa and 221.7 K, respectively. The engine takes 200 kg/s of air and burns 6 kg/s of fuel having a heating value of $Q_R = 43,000\ \ \mathrm{kJ/kg}$), the jet speed is 600 m/s, and exhaust pressure and temperature are 750 K and 30 kPa, respectively.

Calculate the specific thrust ($\frac{T}{\dot{m}_a}$), the thrust specific fuel consumption (*TSFC*), propulsive efficiency (η_{pr}), and thermal efficiency (η_{th}).

6.13 A turbojet engine propels an aircraft flying at a Mach number of 0.9 at an altitude of 11 km. Engine has the following data:

Stagnation temperature rise through the compressor = 190 K.
Fuel heating value = 42.8 MJ/kg.
Turbine inlet temperature (TIT) = 1300 K.
Flight speed = half exhaust speed.
Efficiencies: $\eta_d = 0.9,\ \eta_c = 0.8,\ \eta_{cc} = 0.97,\ \eta_t = 0.84, \eta_n = 0.97, \eta_m = 0.99$
.

Assume variable specific heat ratio ($\gamma_c = 1.4,\ \gamma_h = 4/3, R = 287\ \ \mathrm{J/(kg.K)}$).

Calculate

(A) Compressor pressure ratio
(B) Fuel-to-air ratio
(C) Exhaust Mach number

6.14 The following data apply to a turbojet flying at an altitude where the ambient conditions are 38.25 kPa and 239.4 K.

- Speed of the aircraft: 900 km/h.
- Compressor pressure ratio: 6:1.
- Heat of reaction of the fuel: 43 MJ/kg.
- Turbine inlet temperature: 1400 K.
- Nozzle is choked.
- Nozzle outlet area 0.1 m^2.

Assuming $\gamma_c = 1.4,\ \gamma_h = 4/3,\ R = 0.287\ \ \mathrm{kJ/kg.K}$, *calculate*:

- Inlet area
- Thrust force
- *TSFC*

6.15 A turbojet engine inducts 50 kg of air per second and propels an aircraft with a flight speed of 900 km/h. The fuel-to-air ratio is 0.012 and the heating value of the fuel is 43 MJ/kg. The enthalpy change for the nozzle is 211.25 kJ/kg. Determine:

- The propulsive efficiency
- The thermal efficiency
- *TSFC*
- The propulsive power

6.16 Figure (6.59) illustrates a turbojet engine with two pitot tubes for measuring total pressures at inlet to compressor (P_{02}) and outlet from turbine (P_{05}). These measurements are next displayed for the flight crew as exhaust pressure ratio (*EPR* defined as: $EPR = \frac{P_{05}}{P_{02}}$).

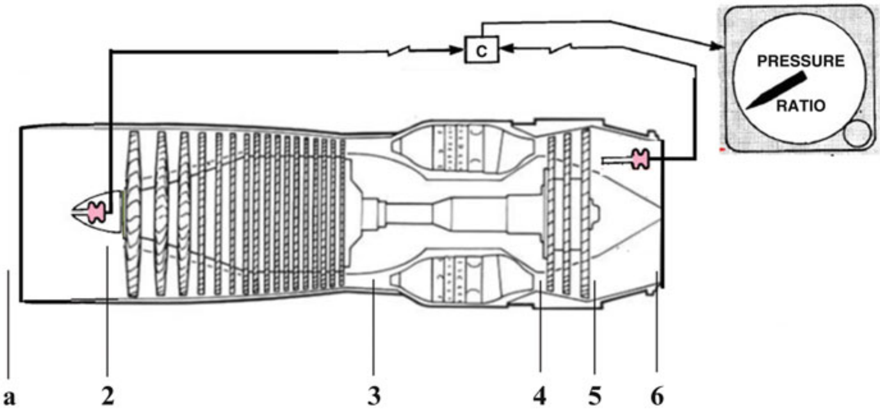


Figure (6.59) Problem (6.16)

Prove that for ideal conditions, exhaust Mach number is expressed by:

$$M_e = \sqrt{\left(\frac{2}{\gamma_h - 1}\right)\left[(EPR)^{\frac{\gamma_h-1}{\gamma_h}}\left(1 + \frac{\gamma_c - 1}{2}M^2\right)^{\left(\frac{\gamma_c}{\gamma_c-1}\right)\left(\frac{\gamma_h-1}{\gamma_h}\right)} - 1\right]} \qquad \text{(A)}$$

Next, if constant properties are assumed ($\gamma_c = \gamma_h = \gamma$), then the above equation is simplified to:

$$M_e = \sqrt{\left(\frac{2}{\gamma - 1}\right)\left[(EPR)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2}M^2\right) - 1\right]} \qquad \text{(B)}$$

Calculate the exhaust Mach number from equation (A) for the case:

$EPR = 1.6$, $M = 2.0$, $\gamma_c = 1.4$, $\gamma_h = 4/3$. Next, recalculate it using the equation (B) for the same data and a constant specific heat ratio $\gamma = 1.4$. *Comment*.

Now consider the case: $M = 2$, $\pi_c = 6.0$, $T_a = 288$ K, $T_{\max} = 1400$ K, $\gamma = 1.4$,

$$Q_R = 43 \ \text{MJ/kg}$$

Calculate

(A) *Exhaust Mach number* using equation (B)
(B) *Turbine* pressure ratio
(C) The *power* delivered to *accessories*

6.17 A. Compare between *ramjet and afterburning turbojet* engines.

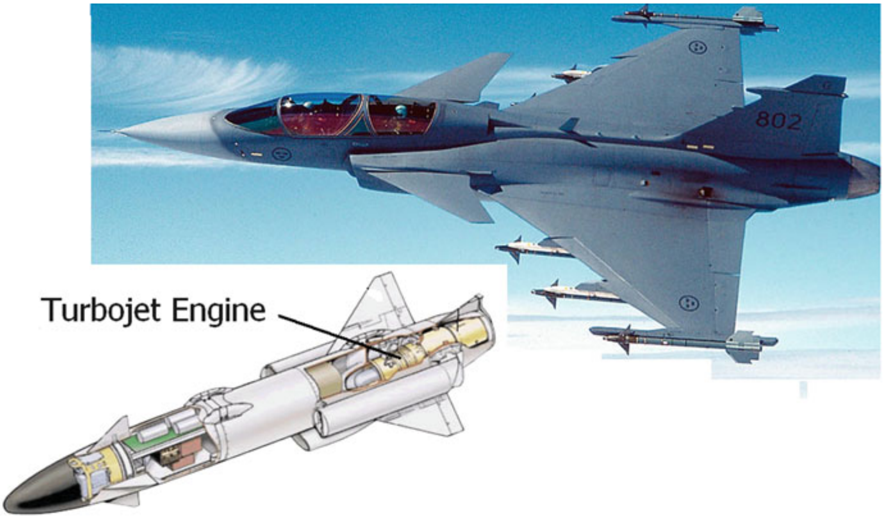


Figure (6.60) Problem (6.17)

B. Gripen multirole fighter (shown in the figure below) is armed with RBS15 MK3 antiship missile which is powered by a French Microturbo TRI 60 engine as shown in Fig. (6.60). It is a *single-spool turbojet* engine with a sustained flight of 0.9 Mach number and nearly sea-level cruise altitude. The engine has the following data:

Thrust force = 4.4 kN	Compressor pressure ratio = 6.3
Air mass flow rate = 8.4 kg/s	*TSFC* = 0.11 kg/(N.hr)
Fuel heating value Q_R = 43 MJ/kg	

Gases expand in nozzle to ambient pressure.

Assuming all processes are ideal and constant values of (γ, C_p), calculate:

(a) Fuel-to-air ratio
(b) Maximum temperature and pressure within the engine
(c) Exhaust speed and Mach number
(d) Inlet area

6.18 The Storm Shadow missile, Fig. (6.61) has been integrated onto the RAF's Tornado GR4 and the Eurofighter Typhoon aircraft. It is a long-range air-to-surface missile that is powered by a single-spool turbojet for sustained flight. For a sea-level cruise operation ($P_a = 101.325$ kPa and $T_a = 298$ K), it has the following data:

Flight Mach number = 0.8.
Thrust = 5.4 kN.
Compressor pressure ratio = 6.3.
Fuel-to-air ratio $f = 0.02$.
Fuel heating value $Q_R = 43\ \text{MJ/kg}$.
$P_e = P_a$.
All processes are ideal.
Constant values of (γ, C_p).

Calculate

1. Maximum temperature and pressure within the engine
2. Exhaust speed and Mach number
3. Air mass flow rate
4. Inlet area
5. *TSFC*

Figure (6.61) Eurofighter Typhoon aircraft and Storm Shadow missile

6.19 A turbojet is flying with a velocity of 300 m/s at an altitude of 9150 m, where the ambient conditions are 32 kPa and −32 °C. The pressure ratio across the compressor is 14, and the temperature at the turbine inlet is 1400 K. Air enters the compressor at a rate of 45 kg/s, and the jet fuel has a heating value of 42,700 kJ/kg. Assuming ideal operation for all components and constant specific heats for air at room temperature, determine:

(a) The temperature and pressure at the turbine exit
(b) The velocity of the exhaust gases
(c) The propulsive power developed
(d) The propulsive efficiency
(e) The rate of fuel consumption

Reexamine this engine if the following efficiencies are considered:

$$\eta_d = 0.92,\ \eta_c = 0.88,\ \eta_b = 0.98,\ \eta_t = 0.9, \eta_n = 0.96$$

What will be the new propulsive power and the rate of fuel consumption?

6.20 Compare the specific fuel consumption of a turbojet and a ramjet which are being considered for flight at M = 1.5 and 17,000 m altitude (ambient conditions of pressure and temperature, 8.786 kPa and 216.7 K, respectively). The turbojet pressure ratio is 8 and the maximum allowable temperature is 1173 K. For the ramjet the maximum temperature is 2300 K. For simplicity ignore aerodynamic losses in both engines. Conventional hydrocarbon fuels are to be used (heating value 43 MJ/kg). Assume $\gamma = 1.4$ and $C_p = 1.005$ kg/(kg.K).

6.21 A turbojet engine propels an aircraft at 300 m/s. The heating value of the fuel is 44 MJ/kg, the speed ratio $\frac{V_f}{V_j} = 0.5$, and the fuel-to-air ratio $\frac{\dot{m}_f}{\dot{m}_a} = 0.02$. Determine:

(a) The ideal propulsion efficiency
(b) The thermal efficiency
(c) The overall efficiency

6.22 A twin-engine turbojet airplane flies at M = 0.8. It has the following data:
$\eta_d = 0.9$, $\eta_c = 0.92$, $\eta_t = 0.95$, $r_b = 0.95$, $\eta_n = 0.96$, $\pi_c = 11$, $TIT = 1400$ K, $Q_R = 42{,}000$ kJ/kg
$T_a = 15$ °C, $P_a = 1$ atm., $\gamma = 1.4$, $C_p = 1.1$ kJ/(kg.K), $L/D = 5$, Airplane Weight $W = 15$ t

Calculate

- The stagnation pressures and temperatures at each engine station
- The thrust per engine
- *TSFC*

6.23 In a turbojet engine, prove that for ideal cycle the specific thrust is expressed by the relation:

$$\frac{T}{\dot{m}_a} = a_0\left\{\sqrt{\frac{2}{\gamma-1}\frac{\tau_\lambda}{\tau_r\tau_c}(\tau_r\tau_c\tau_t-1)} - M_0\right\}$$

$$\text{Where :}\qquad \tau_r = \frac{T_{02}}{T_a},\ \tau_c = \frac{T_{03}}{T_{02}},\ \tau_\lambda = \frac{T_{04}}{T_a}$$

Next, consider the case of removing the turbine driving the compressor and replacing it by an electric motor, as shown in Fig. (6.62). For an ideal approximation to this new engine, assume that compression and expansion are isentropic, and approximate the effect of combustion as heat addition at constant stagnation (total) pressure. For this model, show the specific thrust is given by:

$$\frac{T}{\dot{m}_a} = a_0\left\{\sqrt{\frac{2}{\gamma-1}\frac{\tau_\lambda}{\tau_r\tau_c}(\tau_r\tau_c-1)} - M_0\right\}$$

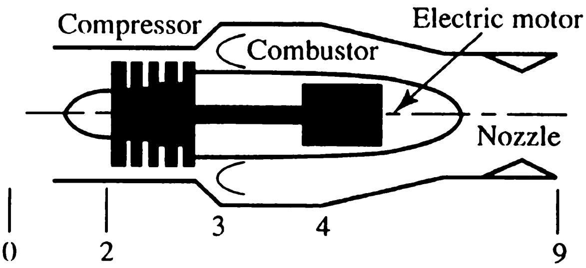


Figure (6.62) Turbojet innovation

Determine the specific thrust for both engines in the following case:

$$T_a = 288\ \text{K},\ M_0 = 0.2,\ T_{0\,\text{max}} = 1600\ \text{K},\ \pi_c = 4,\ \gamma = 1.38,\ C_p = 1.042\ \text{kJ/kg.K}$$

What are your comments?

6.24 Compare between the following three engines: turbojet, turbojet with afterburner, and ramjet. Their operating conditions and common features are:

Altitude range: sea level – 40,000 ft.
Mach number range: 0.0–2.0.
Fuel heating value = 43,260 kJ/kg.
Nozzle is convergent.
Intake inlet diameter = 0.6 m.

Typical turbojet engine (without afterburner)

Fuel mass flow rate = air bleed mass flow rate $\left(\dot{m}_f = \dot{m}_b\right)$.
Maximum fuel-to-air ratio = (stoichiometric ratio/3) or ($f_{\max} \leq 0.068/3$):

$$\Delta P_d = 0, \quad \Delta P_{cc} = 3\%$$
$$\eta_d = \eta_b = \eta_n = 100\%, \quad \eta_c = 87\%, \quad \eta_t = 90\%$$
$$\pi_c = 5.0, \quad TIT = 1100 \text{ K}$$
$$W_c = W_t$$

Turbojet engine with afterburner

Same data as above, together with the following data:

$$f_{ab} = 0.068 - f_{cc}$$
$$\eta_{ab} = 100 \ \%$$
$$\Delta P_{ab} = 4 \ \%$$

Ramjet engine

$$f_{cc} = 0.068$$
$$\Delta P_d = \Delta P_{cc} = 0 \ \%$$
$$\eta_d = \eta_b = \eta_n = 100\%$$
$$(v_{\text{inlet}})_{cc} = 175 \text{ ft/s}$$

Plot the following relations for the three engines:

(a) f_{total}, T_{05}/T_{02}, P_{05}/P_{02}, F_j/A_{front}, F_n/A_{front} versus Mach number (from 0.0 to 2.0)
(b) Exhaust nozzle area versus Mach number (from 0.0 to 2.0)
(c) Exhaust nozzle area for Mach number = 0.8 versus altitude (from 0.0 to 40,000 ft)
(d) F_n/A_{front} versus altitude (from 0.0 to 40,000 ft)
(e) *TSFC* versus altitude (from 0 to 40,000 ft)
(f) *TSFC* versus Mach number (from 0.0 to 2.0)

Turbofan

6.25 The following data apply to a twin-spool turbofan engine, with the fan driven by the LP turbine and the compressor by the HP turbine. Separate hot and cold nozzles are used:

- Overall pressure ratio: 20.0
- Fan pressure ratio: 1.6

- Bypass ratio: 3.5
- Turbine inlet temperature: 1300 K
- Air mass flow: 120 kg/s
- Find the sea-level static thrust and *TSFC* if the ambient pressure and temperature are 1 bar and 288 K. Heat value of the fuel: 43 MJ/kg

6.26 A twin-spool mixed turbofan engine operates with an overall pressure ratio of 18. The fan operates with a pressure ratio of 1.45 and the bypass ratio of 5.0. The turbine inlet temperature is 1400 K. The engine is operating at a Mach number of 0.85 at an altitude where the ambient temperature and pressure are 223.2 K and 0.2645 bar.

Calculate

- The thrust
- *TSFC*
- Propulsive efficiency

6.27 The shown block diagram illustrated in Fig. (6.63) represents a single-spool mixed afterburning turbofan engine flying at altitude 24,000 ft and Mach number 1.8. The fan pressure ratio and mass flow rate are 1.8 and 90 kg/s, respectively. Compressor pressure ratio and mass flow rate are 8.0 and 45 kg/s. Tubine inlet temperature and maximum engine temperature are 1500 and 2600 K. Fuel heating value is 45,000 kJ/kg. Nozzle is of the convergent–divergent type.

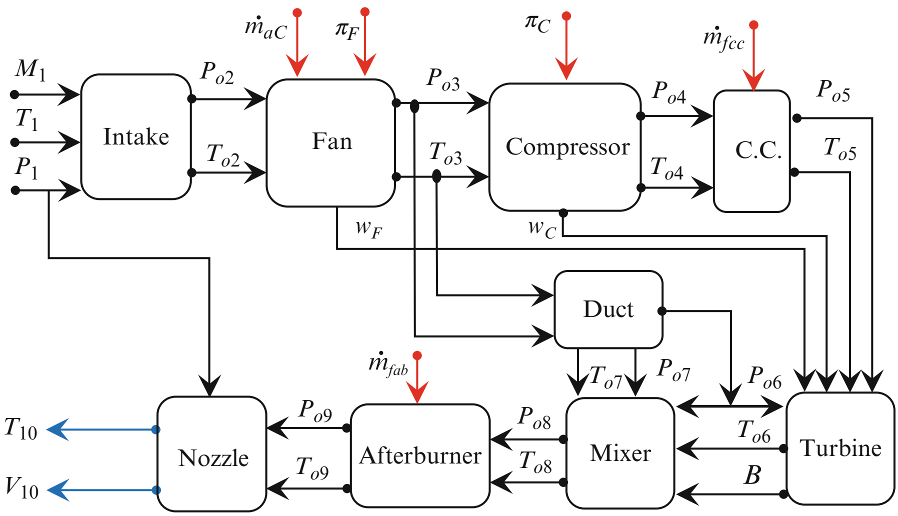


Figure (6.63) Block diagram for a single-spool mixed afterburning turbofan engine

No pressure drop is assumed in combustor, mixer, or afterburner. All other processes are assumed ideal. It is required to:

1. Draw a schematic diagram for engine showing different states
2. Plot T-s diagram for the cycle
3. Calculate bypass ratio
4. Calculate fuel-to-air ratios in combustor and afterburner
5. Calculate the thrust and *TSFC*
6. Calculate the propulsive, thermal and overall efficiencies

6.28 Illustrate how the propulsion efficiency and SFC vary with the flight Mach number for:

1. A conventional turbofan
2. A geared fan with ultrahigh-bypass ratio
3. An unducted fan

Explain the trends in the curves.

6.29 Figure (6.64) shows a fighter airplane propelled by a low-bypass ratio *mixed* afterburning turbofan engine during takeoff from an air carrier. Temperature and pressure of ambient air are 290 K and 101 kPa. Air is ingested into the engine intake at a Mach number of 0.2. Turbofan engine has the following data:

Fan pressure ratio $\pi_f = 1.8$.
Compressor pressure ratio $\pi_c = 5$.
Fuel-to-air ratio in combustion chamber $f = 0.02$.
Afterburner fuel-to-air ratio $f_{ab} = 0.04$.
Fuel heating value $Q_R = 45\ \text{MJ/kg}$.
Percentages of power extracted by fan and compressor from the driving turbines are respectively $\lambda_1 = \lambda_2 = 0.8$.
Assume ideal processes and variable properties: (γ, C_p).

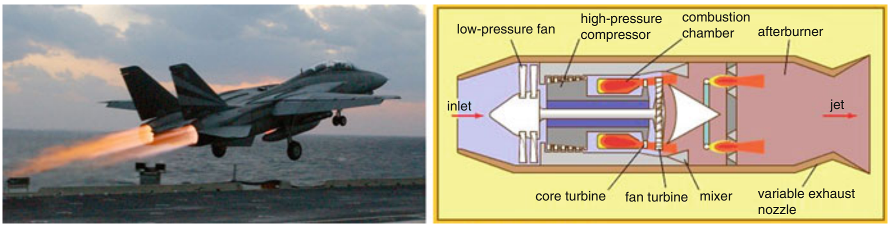


Figure (6.64) Fighter airplane powered by a low-bypass ratio mixed afterburning turbofan engine

Calculate:

1. Bypass ratio
2. Jet speed
3. *TSFC*

6.30 A. Classify turbofan engines

B. Compare between *turbojet and turbofan* engines (maximum Mach number, ceiling, noise, fuel consumption)

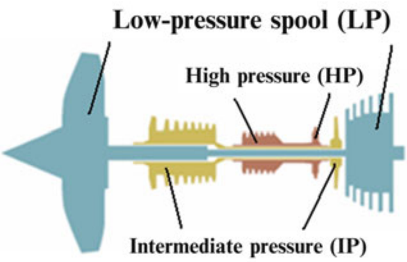


Figure (6.65) Antonov An-225 aircraft (carrying space shuttle) and powered by six engines (Progress D-18T triple-spool unmixed high-bypass ratio turbofan)

C. The shown figure illustrates Antonov An-225 aircraft powered by six engines (Progress D-18T *triple-spool unmixed high-bypass ratio turbofan engine)*. Bleed air is 8 % extracted from HP compressor to cool HP turbine (5 %) and IP turbine (3 %). It has the following data:

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
η_d	0.88	$\Delta p_{c.c}$	0.03	π_F	1.35	λ_1	0.84
η_f	0.9	$\Delta p_{\text{fanduct}}$	0.0	π_{IPC}	5.0	$\lambda_2 = \lambda_3$	1.0
η_c	0.88	cp_c (J/kg.K)	1005	π_{HPC}	4.1	η_n	0.96
η_b	0.98	c_{ph} (J/kg.K)	1148	β	5.7	η_m	0.99
η_t	0.92	R (J/kJ/K)	287	TIT (K)	1600	b	8 %
γ_c	1.4	γ_h	4/3				

For *takeoff conditions (M = 0.2) at* sea level, calculate:

1. The pressure and temperature at outlet of cold and hot nozzles
2. Specific thrust and thrust specific fuel consumption

6.31 A double-spool turbofan engine; Fig. (6.66), is used to power an aircraft flying at speed of 250 m/s at an altitude of 11,000 m. As shown in the figure below, the low-pressure turbine drives the fan and low-pressure compressor, while the high-pressure turbine drives the high-pressure compressor. The engine has the following data:

- Bypass ratio = 8.
- Total ingested air flow rate = 180 kg/s.
- Overall pressure ratio OPR = 35.
- Fan pressure ratio = 1.6.

- Pressure ratio of high-pressure compressor is four times that of the low-pressure compressor; $\pi_{HPC} = 4\ \pi_{LPC}$.
- Turbine inlet temperature = 1650 K.
- Fuel heating value = 43 MJ/kg.

Assuming all processes are ideal and neglecting any pressure drop, *it's required to*:

1. Find the thrust, *TSFC*, and efficiencies of the engine
2. Plot the velocity and temperature distribution over the engine cross section (rear end)

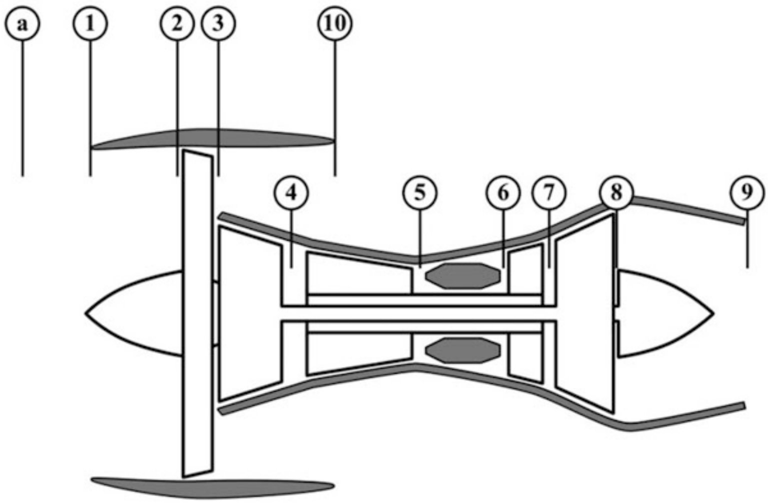


Figure (6.66) Double spool high bypass ratio turbofan engine

6.23 A double-spool turbofan engine is used to power an aircraft flying at speed of 250 m/s at an altitude of 11,000 m. As shown in Fig. (6.67), the low-pressure turbine drives the fan and low-pressure compressor, while the high-pressure turbine drives the high-pressure compressor. Inlet and outlet temperature and velocity of engine are plotted beside the engine layout. The engine has the following data:

- Bypass ratio = 8.
- Total ingested air flow rate = 180 kg/s.
- Overall pressure ratio OPR = 35.
- Pressure ratio of high-pressure compressor is four times that of the low-pressure compressor; $\pi_{HPC} = 4\ \pi_{LPC}$.
- Fuel heating value = 43 MJ/kg.

Assuming all processes are ideal and neglecting any pressure drop, *it's required to find*:

1. Whether the nozzles are choked or not
2. Fan pressure ratio
3. TIT
4. The thrust force

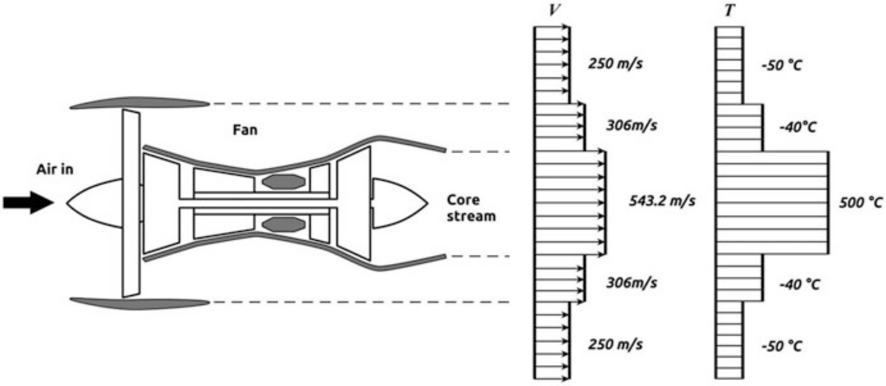


Figure (6.67)

6.33 A single-spool mixed afterburning turbofan engine is to be compared with a single-spool afterburning turbojet engine in its preliminary design stages (Fig. 6.68). *The following characteristics are to be considered:*

- *Ideal* processes, $M = 1.2$
- Altitude = 8 km (where $T_a = 236.23$ K and $P_a = 35.651$ kPa): $c_{p_c} = 1.005$ kJ/kg/K, $\gamma_c = 1.4$, $c_{P_h} = 1.148$ kJ/kg/K, and $\gamma_h = \frac{4}{3}$
- Fuel heating value $Q_R = 43{,}000$ kJ/kg
- Total air flow rate of both engines: $m_a^o = 60$ kg/s

For turbojet

Compressor pressure ratio ($\pi_c = 20$), turbine inlet temperature ($T_{04} = 1400$ K), maximum cycle temperature ($T_{06} = 2200$ K), and unchoked nozzle ($p_7 = p_a$)

For turbofan

Fan pressure ratio ($\pi_f = 2$), compressor pressure ratio ($\pi_c = 10$), turbine inlet temperature ($T_{05} = 1400$ K), maximum cycle temperature ($T_{09} = 2200$ K), and unchoked nozzle ($p_{10} = p_a$)

Calculate

1. Thrust force
2. Propulsive efficiency

3. Thermal efficiency
4. Overall efficiency

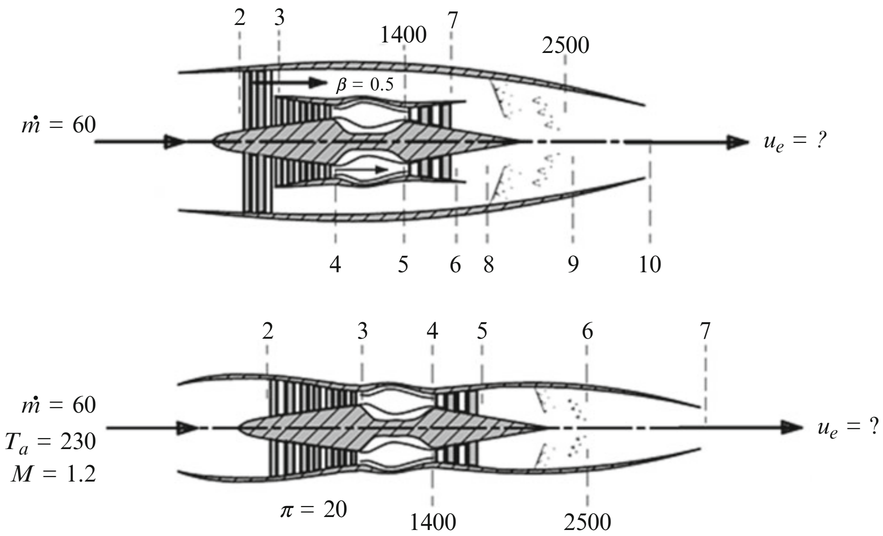


Figure (6.68) Turbojet/Turbofan Layouts

Turboramjet

6.34 The drag of a hypersonic future businesses jet is to be simplified by the following equations:

Flight Mach number	Drag (N)
$M = 0 - 0.75$	$D = 10^5.(4.1\,M^2 - 1.1\,M + 2.1)$
$M = 0.75 - 1.25$	$D = 10^5.(-9.7\,M^2 + 20\,M - 5.9)$
$M = 1.25 - 6$	$D = 10^5.\left(\frac{21}{M^4} - \frac{37}{M^3} + \frac{26}{M^2} - \frac{5.9}{M} + 2.3\right)$

Plot the drag versus Mach number

6.35 If that aircraft in (6.34) is to be driven by two turbojet engines with after-burner, each has the following parameters:

$$TIT = 1800 \text{ K}, \; T_{\max} = 2600 \text{ K}, \quad \pi_0 = 12, \quad \eta_d = 0.9, \quad \eta_c = 0.91, \quad \eta_b = 0.99, \quad \eta_t = 0.92$$

$\eta_n = 0.96$, $\eta_m = 0.99$, $\Delta P_{cc} = 2\ \%$, $\Delta P_{AB} = 3\ \%$. Fuel is hydrocarbon fuel having a heating value of 45.0 MJ/Kg.

Calculate and Plot

– The air mass flow rate (m_a^o)
– The fuel flow rate (m_f^o)
– The thrust specific fuel consumption (*TSFC*)

The afterburner is on for a Mach range of M = 0 to M = 4.
Note: the airplane is designed to flay with dynamic pressure 50,000 N/m^2.

6.36 The aircraft of problem (6.34) is to be driven by two ramjet engine having the following parameters:

$$T_{\max} = 2600\ \text{K}, \quad \eta_d = 0.9, \quad \eta_b = 0.99, \quad \eta_n = 0.98, \quad \Delta P_{cc} = 2.5\ \%,$$

Calculate and Plot

– The air mass flow rate (m_a^o)
– The fuel flow rate (m_f^o)
– The thrust specific fuel consumption (*TSFC*)

The engine is using also hydrocarbon fuel with heating value of 45 MJ/kg.

6.37 The aircraft of problem (6.34) is to be driven by an over–under turboramjet configuration that is constructed of the turbojet in problem (6.34) and the ramjet in problem (6.34) and work on the following regimes.

Regime	Engine condition
$M = 0$ to $M = 0.5$	Turbojet with AB off
$M = 0.5$ to $M = 1.5$	Turbojet with AB on
$M = 1.5$ to $M = 3$	Turbojet with AB off
	Mass flow in turbojet decreasing from 100 %
	m_a^o at $M = 1.5$–0 % m_a^o at $M = 2.5$
	Ramjet is operating as a support for the turbojet engine
$M = 3$ to $M = 6$	The turbojet is off
	The ramjet is driving the A/C alone

Calculate and Plot

– The air mass flow rate (m_a^o)
– The fuel flow rate (m_f^o)
– The thrust specific fuel consumption (*TSFC*)

Comment!

6.38 A wraparound turboramjet engine has the following data when the turbojet engine was operating and the afterburner was lit at an altitude of 60,000 ft.

Flight Mach number	2.5
Turbine inlet temperature	1300 K
Maximum temperature	2000 K
Air mass flow rate	60 kg/s

If the pilot is planning to switch operation to ramjet and stop the turbojet, what is the procedure he has to follow for this transition time?

If the maximum temperature of the ramjet is 2500 K, what will be the needed air mass flow rate to keep the same thrust as the turbojet mode?

Do you expect a dual mode is visible? Explain why or why not.

6.39 Compare between turboramjet and scramjet engines with respect to the following points:

- Flight altitude
- Maximum Mach number
- Fuel consumption

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